Special and General Relativity (PHZ 4601/5606) Fall 2018 Solutions

Set 1

1. Invariance of 2D scalar product $\vec{r}_0 \cdot \vec{r}_1$ under rotation.

Consider the 2D Euclidean rotation

$$\begin{pmatrix} x'^{\,1} \\ x'^{\,2} \end{pmatrix} = \begin{pmatrix} \cos(\phi) & \sin(\phi) \\ -\sin(\phi) & \cos(\phi) \end{pmatrix} \begin{pmatrix} x^1 \\ x^2 \end{pmatrix} = \begin{pmatrix} \cos(\phi)\, x^1 + \sin(\phi)\, x^2 \\ -\sin(\phi)\, x^1 + \cos(\phi)\, x^2 \end{pmatrix} .$$

Therefore, we have

$$\begin{aligned} \vec{r}_0{}' \cdot \vec{r}_1{}' = &\begin{pmatrix} \cos(\phi)\, x_0^1 + \sin(\phi)\, x_0^2 \\ -\sin(\phi)\, x_0^1 + \cos(\phi)\, x_0^2 \end{pmatrix} \cdot \begin{pmatrix} \cos(\phi)\, x_1^1 + \sin(\phi)\, x_1^2 \\ -\sin(\phi)\, x_1^1 + \cos(\phi)\, x_1^2 \end{pmatrix} \\ &= \cos(\phi)^2\, x_0^1\, x_1^1 + \cos(\phi)\, \sin(\phi)\, x_0^1\, x_1^2 + \sin(\phi)\, \cos(\phi)\, x_0^2\, x_1^1 + \sin(\phi)^2\, x_0^2\, x_1^2 \\ &+ \sin(\phi)^2\, x_0^1\, x_1^1 - \sin(\phi)\, \cos(\phi)\, x_0^1\, x_1^2 - \cos(\phi)\, \sin\phi)\, x_0^2\, x_1^1 + \cos(\phi)^2\, x_0^2\, x_1^2 \\ &= x_0^1\, x_1^1 + x_0^2\, x_1^2 \,. \end{aligned}$$