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## Special and General Relativity (PHZ 4601/5606) Fall 2018 Solutions Set 1

## 3. Sum of angles in a geodesic triangle on a sphere.

In the notation of Fig. 8.4 the area of the sphere is  $A_S = 4\pi a^2$  and, consequently, for the half-sphere  $A_{S/2} = 2\pi a^2$ . The area of the triangle with  $r = a \pi/2$  is then

$$A_{\theta} = \frac{\theta}{2\pi} A_{S/2} = \theta a^2 .$$

Setting now  $A_{\theta} = A_S/n = 4\pi a^2/n$  we get

$$\theta = \frac{4\pi}{n} \, .$$

Each of the other two angles has the values  $\pi/2$ . So we get for the sum of the angles in this triangle:

$$\sum_{\text{triangle}} \text{angles} = \pi + \theta = \pi + \frac{4\pi}{n} = \pi \left(1 + \frac{4}{n}\right).$$

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