Special and General Relativity (PHZ 4601/5606) Fall 2018 Solutions

Set 3

5. Gravitational frequency shift.

Close to the earth surface we have

$$\frac{G\,M_E}{R_E} - \frac{G\,M_E}{R_E + h} = \frac{G\,M_E\,h}{R_E} + O\left(R_E^{-2}\right) .$$

As $h \ll R_E$ holds, $g\,h$ is a good approximation. The following Fortran program returns a time difference of 5.5 micro seconds due to the gravitational frequency shift.

```
      program gfs ! BB May 12 2017.
c Gravitational frequency shift due to h=5320 [ft].
      implicit real*8 (a-h,o-z)
c Speed of light in [m/s], feet [ft] in [m], g in [m/s^2]:
      parameter(iuo=6,c=3.d08,ft=0.3d00,g=9.8d00)
c Year in [s], height difference h [m]:
      parameter(y=365.d0*24.d0*3600.d0,h=5320*ft)
      write(iuo,'(/," h,g =",2f9.2,/)') h,g
      dPhi=g*h
      rdf=exp(dPhi/c**2)-1.d00
      rdf1=dPhi/c**2 ! first order Taylor expansion of exp(dPhi/c^2).
      write(iuo,'(" ratio df/f =",1g13.5)') rdf
      write(iuo,'(" 1. order   =",1g13.5,/)') rdf1
      dt=rdf*y
      write(iuo,'(" Time difference over one year =",1g13.5)') dt
      dt=1.d06*dt
      write(iuo,'(" In microseconds               =",1f5.1,/)') dt
      stop "gfs: all done."
      end
```

```
 h,g =  1596.00     9.80

 ratio df/f =  0.17386E-12
 1. order   =  0.17379E-12

 Time difference over one year =  0.54829E-05
 In microseconds               =  5.5
```