Special and General Relativity (PHZ 4601/5606) Fall 2018 Solutions Set 3

7. Relativistic distance measurements by light signals.

For 2,757,789,531,312 counts of the Cesium clock the elapsed time is 300 $[s]$.

(1) The relation $x = (\triangle t/2)\, c$ gives for all three times

$$10^{-3}\,[s] \times 3 \times 10^5\,[km/s] \;=\; 300\,[km]\,.$$

Therefore, O_2 is at rest with respect to O_1.

(2) We find the following positions at the following times:

$$x_1 = 300\,[km] \;\text{ at }\; t_1 = 1.001\,[s]\,,$$
$$x_2 = 600\,[km] \;\text{ at }\; t_2 = 2.002\,[s]\,,$$
$$x_3 = 900\,[km] \;\text{ at }\; t_3 = 3.003\,[s]\,.$$

This gives the velocities

$$v_{21} = \frac{x_2 - x_1}{t_2 - t_1} \;=\; \frac{300\,[km]}{(2.002 - 1.001\,[s]} \;=\; \frac{300\,[km]}{1.001\,[s]} \;\approx\; 299.7\,[km/s]\,,$$
$$v_{32} = \frac{x_3 - x_2}{t_3 - t_2} \;=\; \frac{300\,[km]}{(3.003 - 2.002)\,[s]} \;=\; \frac{300\,[km]}{1.001\,[s]} \;\approx\; 299.7\,[km/s]\,.$$

So, the results are consistent with the idea that O_2 is at rest in an inertial frame, which moves with about $299.7\,[km/s]$ with respect to the inertial frame of O_1. The position of O_2 with respect to O_1 is then given by

$$x(t) \;=\; x_0 + v\,t \;=\; \frac{300\,[km]}{1.001\,[s]}\,t\,,$$

where $x_0 = 0$ followed from $x(t_1) = 300\,[km]$ for $t_1 = 1.001\,[s]$.

(3) We find again the positions $x_1 = 300\,[km]$ at $t_1 = 1.001\,[s]$ and $x_2 = 600\,[km]$ at $t_2 = 2.002\,[s]$, which give $v_{21} = (300/1.001)\,[km/s]$, but now

$$x_3 = 1200\,[km] \;\text{ at }\; t_3 = 3.004\,[s]\,,$$

which gives

$$v_{32} = \frac{x_3 - x_2}{t_3 - t_2} \;=\; \frac{600\,[km]}{1.002\,[s]} \;\approx\; 598.2\,[km/s]\,.$$

As v_{21} and v_{32} disagree, O_2 cannot be at rest in an inertial frame.