Special and General Relativity (PHZ 4601/5606) Fall 2018 Solutions

Set 13

32. Show Eq. (11.4), p.229 of Rindler.

The starting point is the appendix, p.419, 420, of Rindler or, better, its shortened version posed on the Web. We choose the identification

$$\begin{aligned} d\vec{s}^{\,2} &= a\,(dx^1)^2 + b\,(dx^2)^2 + (dx^3)^2 + (dx^4)^2 \\ &= a\,(dt)^2 + b\,(dr)^2 + c\,(d\theta)^2 + d\,(d\phi)^2\,, \end{aligned}$$

respectively, with $a = \exp[+A(r)]$, $b = -\exp[+B(r)]$, $c = -r^2$, $d = -r^2\sin^2\theta$. The non-zero derivatives are the

$$' = \frac{\partial}{\partial\, x^2} = \frac{\partial}{\partial\, r} \ \text{ and, on } d \text{ only, } \frac{\partial}{\partial\, x^3} = \frac{\partial}{\partial\, \theta}\,.$$

Therefore, the non-zero contributions to the Ricci tensor are

$$R_{tt} = R_{11} = \beta\, a_{22} - \beta\, a_2\,(\alpha\, a_2 + \beta\, b_2 - \gamma\, c_2 - \delta_2\, d_2)\,,$$

where α, β, γ, δ are defined by $\alpha = 1/2a$, $\beta = 1/2b$, $\gamma = 1/2c$, $\delta = 1/2d$. We have

$$a' = A' \exp(+A)\,, \quad b' = -B' \exp(+B) \quad c' = -2r \quad d' = -2r\,\sin^2\theta$$

and $a'' = A'' \exp(A) + (A')^2 \exp(A)$. Inserting these gives

$$\begin{aligned} R_{tt} &= -\exp(A-B)\,\frac{1}{2}\,\left(A'' + A'^2\right) \\ &\quad + \exp(A-B)\,\frac{1}{2}\,A'\,\left(\frac{1}{2}\,A' + \frac{1}{2}\,B' - \frac{1}{r} - \frac{1}{r}\right) \\ &= -\exp(A-B)\,\left(\frac{1}{2}\,A'' - \frac{1}{4}\,A'\,B' + \frac{1}{4}\,A'^2 + A'/r\right)\,. \end{aligned}$$