

## From the Appendix of Rindler Ricci tensor for a diagonal metric.

The appendix of Rindler (p.419-421) deals with the metric

$$d\vec{s}^2 = a(dx^1)^2 + b(dx^2)^2 + c(dx^3)^2 + d(dx^4)^2, \quad (1)$$

where  $a, b, c, d$  are functions of the coordinates. Further notation is

$$\alpha = \frac{1}{2a}, \quad \beta = \frac{1}{2b}, \quad \gamma = \frac{1}{2c}, \quad \delta = \frac{1}{2d}, \quad (2)$$

and

$$a_i = \frac{\partial a}{\partial x^i}, \quad a_{ij} = \frac{\partial^2 a}{\partial x^i \partial x^j}, \quad \text{and so on,} \quad (3)$$

where we allow  $i, j = 1, 2, 3, 4$ . From the definitions of the Christoffel symbols

$$\Gamma_{ijk} = \frac{1}{2} (g_{ij,k} + g_{ik,j} - g_{jk,i}), \quad \Gamma^i_{jk} = g^{ih} \Gamma_{hjk} \quad (4)$$

[Rindler (10.13) and (10.14), p.206] one finds

$$\Gamma^1_{23} = 0, \quad \Gamma^1_{22} = -\alpha b_1, \quad \Gamma^1_{1i} = \alpha a_i. \quad (5)$$

From these three typical  $\Gamma$ s all other can be obtained by the obvious permutations. For example,  $\Gamma^2_{33} = -\beta c_2, \Gamma^4_{44} = -\delta d_2$ .

After calculating the Riemann curvature tensor first, the Ricci tensor  $R_{ij}$  as defined by Rindler (10.68), p.219 becomes

$$\begin{aligned} R_{11} = & + \beta a_{22} + \gamma a_{33} + \delta a_{44} \\ & + \beta b_{11} + \gamma c_{11} + \delta d_{11} \\ & - \beta^2 b_1^2 - \gamma^2 c_1^2 - \delta^2 d_1^2 \\ & - \alpha a_1 ( + \beta b_1 + \gamma c_1 + \delta d_1 ) \\ - \beta a_2 ( & \alpha a_2 + \beta b_2 - \gamma c_2 - \delta d_2 ) \\ - \gamma a_3 ( & \alpha a_3 - \beta b_3 + \gamma c_3 - \delta d_3 ) \\ - \delta a_4 ( & \alpha a_4 - \beta b_4 - \gamma c_4 + \delta d_4 ) \end{aligned} \quad (6)$$

$$\begin{aligned} R_{12} = & + \gamma c_{12} + \delta d_{12} \\ & - \gamma^2 c_1 c_2 - \delta^2 d_1 d_2 \\ & - \alpha \beta a_2 c_1 - \alpha \delta a_2 d_1 \\ & - \beta \gamma b_1 c_2 - \beta \delta b_1 d_2 \end{aligned} \quad (7)$$