Motion in One Dimension

The study of motion is called kinematics. Displacement of a point particle:

$$\triangle x = x_2 - x_1$$

where x_1 is the position at time t_1 and x_2 is the position at time t_2, $t_2 > t_1$. The average velocity, Tipler figure 2-4, is defined by:

$$v_{\mathrm{av}} = \frac{\triangle x}{\triangle t} = \frac{x_2 - x_1}{t_2 - t_1}$$

The average velocity can take positive or negative values. The average speed is defined by:

$$\text{Average speed} = \frac{\text{total distance}}{\text{total time}} = \frac{\triangle s}{\triangle t} \geq 0$$

Instantaneous velocity, Tipler figure 2-5:

$$v(t) = \lim_{\triangle t \to 0} \frac{\triangle x}{\triangle t} = \lim_{t_2 \to t} \frac{x - x_2}{t - t_2} = \frac{d\,x}{d\,t}$$

This is the slope of the tangent of the curve $x(t)$ at t and called derivative. Note: $t = t_1$, $x = x_1$ of the difference notation. The instantaneous speed is the absolute value $|v(t)|$ of the instantaneous velocity.

Relative velocity: If a particle p moves with velocity $v_{\rm pA}$ relative to a coordinate system A, which moves with velocity $v_{\rm AB}$ relative to a coordinate system B, the velocity of p with respect to B is

$$v_{\rm pB} = v_{\rm pA} + v_{\rm AB}$$

This equation is only valid for velocities $|v| \ll c$, where c is the speed of light.

Acceleration: Acceleration is the rate of change of the instantaneous velocity. The average acceleration is defined by:

$$a_{\rm av} = \frac{\triangle v}{\triangle t} = \frac{v_2 - v_1}{t_2 - t_1}$$

The instantaneous acceleration is:

$$a(t) = \lim_{\triangle t \to 0} \frac{\triangle v}{\triangle t} = \lim_{t_2 \to t} \frac{v - v_2}{t - t_2} = \frac{d\,v}{d\,t} = \frac{d^2 x}{dt^2}$$

Motion With Constant Acceleration

$$\frac{d\,v}{d\,t} = a = a_{\rm av}$$

Integration:

$$v(t) = \frac{d\,x}{d\,t} = v_0 + a\,t$$

Here v_0 is the velocity at time zero, called initial condition.

Second integration:

$$x(t) = x_0 + v_0\, t + \frac{1}{2}\, a\, t^2$$

Here x_0 is the second initial condition, the position at time zero.

Illustrations: (a) Air track; (b) constant acceleration of free fall.

Note,

$$\triangle x = x - x_0 = v_{\rm av}\, t = \frac{1}{2}\,(v + v_0)\, t = \frac{1}{2}\,(v + v_0)\, \frac{v - v_0}{a} = \frac{v^2 - v_0^2}{2\, a}$$

$$v^2 = v_0^2 + 2\, a\, \triangle x$$

General integration: Tipler figure 2-18.

Questions (PRS)

Units: The meter is nowadays defined by

1. Scratches on a platinium-iridium alloy kept at the International Bureau of Weights and Measures in Sèvres, France.

2. As a fraction of the circumference of the earth around the equator.

3. As a distance traveled by the speed of light in vacuum during a certain time.

Significant figures: The experimental measurements 2.50 m and 2.496 m

1. Disagree.
2. Are identical.
3. Agree within measurement errors.

Order of magnitude: The number 250 has the order of magnitude

1. 10
2. 10^2
3. 10^3
4. 2.5×10^2

Relative velocity: A swimmer moves a river upstream with the velocity of 1.20 km/h relative to the water. The speed of the water is 0.1 m/s relative to the ground. The velocity of the swimmer over ground (positive upstream) is

1. 0 km/h

2. $+3000$ m/s

3. $+0.84$ km/h

4. -0.84 km/h

5. -3000 m/s

Acceleration: Assume the acceleration of a point particle is $a = 0$. The following holds:

1. $v(t) = v_0$
2. $v(t) = v_0 + t$
3. $v(t) = v_0 \, t$

1. $x(t) = x_0$
2. $x(t) = x_0 + v_0 \, t$
3. $x(t) = x_0 \; + \frac{1}{2} \, v_0 \, t$