Motion in Two and Three Dimension

Displacement Vector:

$$\vec{A}$$

is defined by its magnitude and direction and graphically represented by an arrow. The magnitude is written as

$$|\vec{A}| \text{ or simply } A$$

Addition of displacement vectors:

$$\vec{C} = \vec{A} + \vec{B}$$

by putting the arrows together, Tipler figure 3-1. Note: this does not imply $C = A + B$.

Multiplication by a scalar (number):

$$\vec{B} = s\,\vec{A}$$

Direction unchanged for $s > 0$, inverted for $s < 0$, magnitude $B = |s|\,A$.

Subtracting a vector $\vec{C} = \vec{A} - \vec{B}$: Add the negative vector (direction inverted).

Components of vectors (2D): Tipler, figure 3-9.

$$A_x = A\,\cos(\theta)$$

$$A_y = A\,\sin(\theta)$$

$$A = \sqrt{A_x^2 + A_y^2} \quad \text{Pythagoreas}$$

$$\text{in 3D}: \quad A = \sqrt{A_x^2 + A_y^2 + A_z^2}\ .$$

,

Addition in components:

$$C_x = A_x + B_x$$
$$C_y = A_y + B_y$$
$$C_z = A_z + B_z$$

Unit Vectors: along the coordinate axis, Tipler figure 3-12. Notations of the literature:

$$\widehat{i} = \widehat{x} = \widehat{e}_x \qquad (\mathrm{in}\ x - \mathrm{direction})$$

$$\widehat{j} = \widehat{y} = \widehat{e}_y \qquad (\mathrm{in}\ y - \mathrm{direction})$$
$$\widehat{k} = \widehat{z} = \widehat{e}_z \qquad (\mathrm{in}\ z - \mathrm{direction})$$

Now, every vector can be written like

$$\vec{A} = A_x\,\widehat{x} + A_y\,\widehat{y} + A_z\,\widehat{z}$$

Tipler table 3-1: Overview of Vector Properties.

Position, Velocity and Acceleration

Position vector:

$$\vec{r} = x\,\widehat{x} + y\,\widehat{y} = x\,\widehat{i} + y\,\widehat{j}$$

Change in the position, Tipler figure 3-13:

$$\triangle\vec{r} = \vec{r}_2 - \vec{r}_1$$

Average velocity vector:

$$\vec{v}_{\rm av} = \frac{\triangle\vec{r}}{\triangle t}$$

Instantaneous velocity vector, Tipler figure 3-14:

$$\vec{v} = \lim_{\triangle t \to 0} \frac{\triangle \vec{r}}{\triangle t} = \frac{d\,\vec{r}}{d\,t}$$

Component by component!

$$v_x = \frac{d\,x}{d\,t}\,, \qquad v_y = \frac{d\,y}{d\,t}\,, \qquad v_z = \frac{d\,z}{d\,t}$$

Relative velocity:

$$\vec{v}_{\mathrm{pB}} = \vec{v}_{\mathrm{pA}} + \vec{v}_{AB}$$

Example: Frame A a train and frame B ground.

Average acceleration vector:

$$\vec{a}_{\mathrm{av}} = \frac{\triangle \vec{v}}{\triangle t}$$

Instantaneous acceleration vector:

$$\vec{a} = \lim_{\triangle t \to 0} \frac{\triangle \vec{v}}{\triangle t} = \frac{d\,\vec{v}}{d\,t} = \frac{d^2 \vec{r}}{d\,t^2}$$

Component notation:

$$\vec{r}(t) = \begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix}, \text{ etc.}$$

Projectile Motion

We neglect friction in the following!

Differential equations:

$$\frac{d^2x}{d\,t^2} = \frac{d\,v_x}{d\,t} = a_x = 0$$

$$\frac{d^2y}{d\,t^2} = \frac{d\,v_y}{d\,t} = a_y = -g$$

and

$$g = 9.81\,m/s^2$$

as (approximately) measured in the last lecture.

First integration $\vec{v}(t)$:

$$
\begin{aligned}
v_x &= v_{0x} \\
v_y &= v_{0y} - g\,t
\end{aligned}
$$

Initial velocity $\vec{v}_0$:

$$v_{0x} = v_0 \cos(\theta_0)$$

$$v_{0y} = v_0 \sin(\theta_0)$$

Second integration (solution) $\vec{r}(t)$:

$$\begin{aligned} x(t) &= x_0 + v_{0x}\, t \\ y(t) &= y_0 + v_{0y}\, t - \frac{1}{2}\, g\, t^2 \end{aligned}$$

Possible initial positions: $x_0 = y_0 = 0$.

Tipler-Mosca figure 3-25: Path of a projectile with velocity components.

Illustration: Air track.

Questions on Projectile Motion (PRS)

Consider the 2D orbit given by

$$x(t) = v_{0x}\, t\,,\ \ v_{0x} > 0\,, \qquad y(t) = v_{0y}\, t - \frac{1}{2} g\, t^2\,,\ \ v_{0y} > 0$$

At which time is the highest point reached?

$$1.\quad t_{\max} = \frac{v_{0y}}{g}\,, \qquad 2.\quad t_{\max} = \frac{2\, v_{0y}}{g}$$

At which time is the ground $y = 0$ reached again?

$$1.\quad t_1 = \frac{2\, v_{0x}}{g}\,, \qquad 2.\quad t_1 = \frac{2\, v_{0y}}{g}$$

Consider two canon balls and assume their initial conditions for x_0 and v_{0x} are different, while those for y_0 and v_{0y} are identical. Which of the following holds (pick one): (Demonstration)

1. The two canon balls hit the ground at distinct times.

2. The two canon balls hit the ground at the same time.

3. The provided information on the initial conditions is insufficient to decide between answer 1 and 2.

Two point particles have the following ininital conditions: $x_{10} = x_{20},\ v_{10x} = v_{20x},\ y_{10} = y_{20}$ and $v_{10y} = 0,\ v_{20y} > 0$. Which of the following holds:

1. At time zero the two particles are at different positions.

2. At time zero the two particles are at the same position and they will remain to stay together.

3. At time zero the two particles are at the same position and at all future times their positions are different.

4. At time zero the two particles are at the same position and they will meet once again in the future. Besides this their future positions will be different.

Tipler-Mosca figure 3-31: Shooting a monkey (Demonstration).

The gun is fired at the same instant when the monkey drops. Will the dart hit? (PRS: 1. Yes and 2.No).