Another Projectile Motion Question

The initial conditions of two point particles are as follows:

$$x_{20} = x_{10} + \triangle x,\ \ \triangle x > 0$$

$$y_{20} = y_{10} + \triangle y,\ \ \triangle y > 0$$

$$v_{10x} > 0,\ \ v_{10y} > 0,\ \ \frac{v_{10y}}{v_{10x}} = \frac{\triangle y}{\triangle x}$$

$$v_{20x} = v_{20y} = 0$$

There is free fall downwards in y direction. Which of the following holds (draw the situation and pick one choice)?

1. Particle one will hit particle two unless it hits ground first.

2. Particle one will not hit particle two.

Solution

$$
\begin{aligned}
x_1(t) &= v_{10x}\, t \\
y_1(t) &= v_{10y}\, t - \frac{1}{2}\, g\, t^2 \\
x_2(t) &= \triangle x \\
y_2(t) &= \triangle y - \frac{1}{2}\, g\, t^2
\end{aligned}
$$

At some time t_1:

$$
\begin{aligned}
x_1(t_1) &= v_{10x}\, t_1 = \triangle x = x_2(t_1) \\
y_1(t_1) &= v_{10y}\, t_1 - \frac{1}{2}\, g\, t_1^2 = \frac{\triangle y}{\triangle x}\, v_{10x}\, t_1 \\
&= \triangle y - \frac{1}{2}\, g\, t_1^2 = y_2(t_1) \quad \text{hit!}
\end{aligned}
$$

Newton's Laws

1. Law of inertia. An object continues to travel with constant velocity (including zero) unless acted on by an external force.

2. The acceleration $\vec{a}$ of an object is given by

$$m\,\vec{a} = \vec{F}_{\rm net} = \sum_i \vec{F}_i$$

   where m is the mass of the object and $\vec{F}_{\rm net}$ the net external force.

3. Action = Reaction. Forces always occur in equal and opposite pairs. If object A exterts a force on object B, an equal but opposite force is exterted by object B on A.

Definition of the Mass

Mass is an intrinsic property of an object that measures its resistence to acceleration, that is the object's inertia. If the same force F produces the acceleration a_1 when applied to object 1 and acceleration a_2 when applied to object 2, the ratio of their masses is defined to be

$$\frac{m_2}{m_1} = \frac{a_1}{a_2} \ .$$

By comparing with the 1 kg object kept at Sévres we can thus measure the mass of any object.

Unit of force: The force required to produce the acceleration of $1\,\mathrm{m/s}^2$ on the 1 kg standard object is called one Newton

$$1\,\mathrm{N} = 1\,\mathrm{kg\,m\,/\,s}^2 \ .$$

Weight

The force due to the gravitational field $\vec{g}$ is called weight

$$\vec{w} = m\,\vec{g}\ .$$

Approximately,

$$g = |\vec{g}| = 9.81\,\mathrm{N/kg} = 9.81\,\mathrm{m/s^2}\ .$$

An object is said to be in free fall when gravity is the only force acting on it.

An example of Newton's 3rd law: Block on a table.

Springs

When a spring is compressed or extended by a small amount $\triangle x$, the force it exerts is (Hooke's law)

$$F_x = -k \triangle x \quad \text{with} \quad \triangle x = x - x_0 \ .$$

Figure 4-5 of Tipler-Mosca.

Let us choose $x_0 = 0$, the differential equation for this motion is

$$F_x = m\, a = m \frac{d^2 x}{d\, t^2} = -k\, x \ .$$

It will be solved later in this course.

Solving Problems

1. Draw a diagram.
2. Isolate the object under investigation.
3. Indicate all forces acting on the object.
4. Choose a convenient coordinate system.
5. Decompose the forces into components along the major axes.
6. Use Newton's laws and solve the resulting equations for the unknowns.

Example: Sledge

Tipler-Mosca figures 4-10 and 4-11.

$$\begin{aligned} \vec{F}_n + \vec{w} + \vec{F} &= m\,\vec{a} \\ F_x &= m\,a_x \quad (\text{as } F_{n,x} = 0,\ w_x = 0) \\ F_{n,y} + w_y + F_y &= 0 = m\,a_y \end{aligned}$$

Example: Inclined Plane

Tipler-Mosca figure 4-13.

$$
\begin{aligned}
0 &= F_{n,y} + \omega_y \\
m\,a_x &= w_x = w\,f(\theta) = m\,g\,f(\theta) \\
x(t) &= x_0 + v_0\,t + \frac{1}{2}\,g\,f(\theta)\,t^2 \;.
\end{aligned}
$$

PRS:

1. $f(\theta) = \sin(\theta)$ 2. $f(\theta) = \cos(\theta)$.

Example: String Tension

Tipler-Mosca figure 4-14 (special case) and figure 4-37.

$$\sum \vec{F} = \vec{T}_1 + \vec{T}_2 + \vec{w} = m\,\vec{a} = 0$$

$$\sum F_x = T_1\,\cos(\alpha) - T_2\,\cos(\beta) = 0$$
$$\sum F_y = T_1\,\sin(\alpha) + T_2\,\sin(\beta) - m\,g = 0$$

$$T_2 = T_1\,\frac{\cos(\alpha)}{\cos(\beta)}$$
$$T_1\,\sin(\alpha) + T_1\,\frac{\sin(\beta)\,\cos(\alpha)}{\cos(\beta)} - m\,g = 0$$
$$T_1\,\left[\frac{\sin(\alpha)\,\cos(\beta) + \sin(\beta)\,\cos(\alpha)}{\cos(\beta)}\right] = m\,g$$

$$T_1 \left[\frac{\sin(\alpha+\beta)}{\cos(\beta)}\right] = m\,g$$

$$T_1 = m\,g \left[\frac{\cos(\beta)}{\sin(\alpha+\beta)}\right] \quad \text{and} \quad T_2 = m\,g \left[\frac{\cos(\alpha)}{\sin(\alpha+\beta)}\right]$$

What happens for $\alpha = \beta \rightarrow 0$?

Example: Two Connected Blocks

Tipler-mosca figure 4-21 with $\theta = 0$. For block one gravity is cancelled by the normal force, it remains the effect of gravity acting on block two.

$$F = m_2\, g = (m_1 + m_2)\, a$$

$$a = \frac{m_2}{m_1 + m_2}\, g$$

$$x_1(t) = x_0 + v_0\, t + \frac{1}{2}\, a\, t^2$$

$$y_2(t) = y_0 + v_0\, t - \frac{1}{2}\, a\, t^2$$

Homework: Calculate also the tension in the string.

Questions

Assume $m_1 + m_2 = 1\,\text{kg}$ and $m_2 = 50\,\text{g}$. The expected acceleration is (pick one):

1. $9.81\,\mathrm{m/s^2}$
2. $0.4905\,\mathrm{m/s^2}$
3. $0.005\,\mathrm{m/s^2}$
4. $50\,\mathrm{m/s^2}$
5. $4905\,\mathrm{m/s^2}$

Definition of the mass: Let $\vec{F}_1$ be the force that acts on object one and $\vec{F}_2$ be the force that acts on object two. The masses of the objects are defined by (pick one):

1.
$$\frac{m_2}{m_1} = \frac{a_1}{a_2} \quad \text{for} \quad \vec{F}_1 \neq \vec{F}_2$$

2.
$$\frac{m_2}{m_1} = \frac{a_1}{a_2} \quad \text{for} \quad \vec{F}_1 = \vec{F}_2$$