Mini Exam # 1

- You get them back in the the recitation section for which you are officially enrolled.
- One third of you did very well (≥ 18 points out of 20).
- The average was 13.4. If you stay in average, you will pass.
- If you scored below 10 points, you are in danger for a D or F. Work hard now!
- If you scored zero points or did not show up (and this is a large group too!): You are heading for a straight F. Have no illusions about catch-up possibilities. You may have exactly one more chance. It is our experience that after falling two minis back, catch-ups are in practice almost impossible.

Office Hours Berg

This week (Sep. 15-19) shifted to T R 11 am – 12 am. Next week: at the normal times again.

Tabor and Balicas are this week out-of-town and their office hours are cancelled.

Example: Atwood's Machine

Tipler-Mosca figure 4-58. In the following a masless, frictionless pully and a massless string are assumed.

$$F = (m_1 - m_2)\, g = (m_1 + m_2)\, a$$

$$a = \frac{m_1 - m_2}{m_1 + m_2}\, g$$

Tension in the string:

$$T = m_1\,(g - a) = \left[\frac{m_1\,(m_1 + m_2)}{m_1 + m_2} - \frac{m_1\,(m_1 - m_2)}{m_1 + m_2}\right]\, g$$

$$= \frac{2\, m_1\, m_2\, g}{m_1 + m_2}$$

Questions

With

$$m_1 = 251\,g = 0.251\,kg \quad \text{and} \quad m_2 = 250\,g = 0.250\,kg$$

we measured

$$\triangle t = 11\,s \quad \text{for} \quad \triangle x = 1\,m\ .$$

Approximately, this corresponds to the following lower bound on the gravitational constant g (pick one):

1. $5\,\mathrm{m/s^2}$ 2. $6\,\mathrm{m/s^2}$ 3. $7\,\mathrm{m/s^2}$

4. $8\,\mathrm{m/s^2}$ 5. $9\,\mathrm{m/s^2}$ 6. $10\,\mathrm{m/s^2}$

Solution

Calculate the acceleration a from the our measurements of $\triangle t$ and $\triangle x$,

$$\triangle x = \frac{1}{2}\, a\, t^2 \;\Rightarrow\; a = \frac{2\,\triangle x}{t^2}\,,$$

and solve the equation

$$(m_1 - m_2)\, g = (m_1 + m_2)\, a$$

for a.

Why is the estimate a lower bound?

1. Because of friction.
2. Because of the mass of the pulley.
3. Because of friction and the mass of the pulley.
4. Because of the tension in the rope.
5. Because of friction, the mass of the pulley and the tension in the rope.

Friction (Chapter 5-1 of Tipler-Mosca)

Friction is a complicated phenomenon that arises when the electromagnetic interactions of molecules between two surfaces in close contact lead to a bonding. This bonding is similar to the molecular bonding which keeps objects together.

If an external force acts on a heavy box standing on a floor (see figure 5-1 of Tipler-Mosca), the box may not move because the external force is balanced by the force f_s of static friction. Its maximum value $f_{s,max}$ is obtained when any further increase of the external force will cause the box to slide. To a good approximation $f_{s,max}$ is simply proportional to the normal force

$$f_{s,max} = \mu_s \, F_n$$

where μ_s is called the coefficient of static friction (some approximate values are given in table 5-1 of Tipler-Mosca). If the box does not move we have

$$f_s \leq f_{s,max} \ .$$

Kinetic friction (also called sliding friction): Once the box slides, a constant force is needed to keep it sliding at constant velocity. The opposing force is the force of kinetic friction. In a good approximation it is also simply proportional to the normal force

$$f_k = \mu_k \, F_n$$

where μ_k is called the coefficient of kinetic friction (some approximate values are also given in table 5-1 of Tipler-Mosca).

Experimentally it is found that $\mu_k < \mu_s$. Figure 5-3 of Tipler-Mosca shows the frictional force exerted on the box by the floor as a function of the applied force.

Example: Block on an inclined plane with friction.

Tipler-Mosca figure 5-6:

$$F_n - m \, g \, \cos(\theta) = m \, a_y = 0$$

$$F_n = m \, g \, \cos(\theta)$$

$$m\,g\,\sin(\theta) - f_s = m\,a_x = 0$$
$$f_s = m\,g\,\sin(\theta)$$

For the maximum:

$$= \mu_s\,F_n = \mu_s\,m\;g\,cos(\theta)$$
$$\mu_s = \frac{\sin(\theta)}{\cos(\theta)} = \tan(\theta)$$

Questions

Coefficient of static friction:

We measured

$$\theta_{\rm max} = 30^0$$

for a block of wood on an inclined plane of wood. This correspond to the coefficient of static friction (use your calculator and pick the result)

$\mu_s =$

1. 0.55
2. 0.55 m
3. 0.58
4. 0.58 m
5. 0.61
6. 0.61 m

Example: Forces acting on a car.

How to get the car moving: The car has front-wheel drive and is just starting to move, figure 5-16 of Tipler-Mosca.

The weight is balanced by the normal forces $\vec{F}_n$.

The engine makes the front wheels rotate. What would happen if the road were frictionless?

1. The car would move.

2. The car would not move, the wheels would merely spin.

When the wheel rolls without slipping, the tire thread touching the road is at rest relative to it and the friction between the road and the tire is static friction $f_s \leq \mu_s F_n$, figure 5-17 of Tipler-Mosca.

How to stop the car: The force that brings the car to a stop when it brakes is the force of friction exerted by the road on the tires, figure 5-18 of Tipler-Mosca. When the frictional force exerted by the road is constant, the acceleration is constant (and negative), and the stopping time $\triangle t$ is given by

$$\triangle t = \frac{v_{x,0}}{|a_x|} \ .$$

The distance $\triangle x$ is then the average velocity $v_{av} = v_{x,0}/2$ times the stopping time

$$\triangle x = \frac{v_{x,0} \, \triangle t}{2} \ .$$

Eliminating $\triangle t$ from this equation gives

$$\triangle x = \frac{v_0^2}{2 \, |a_x|}$$

Optimal (ideal) acceleration:

$$a_x = -\frac{f_s}{m} = -\frac{\mu_s\, F_n}{m} = -\frac{\mu_s\, m\, g}{m} = -\mu_s\, g$$

Less ideal, when the wheels slip:

$$a_x = -\mu_k\, g$$

In reality one will normally be in-between those two values.

Circular Motion (Chapter 3-5 of Tipler-Mosca)

Centripetal Acceleration:

Pythagorean theorem:

$$(r+h)^2 = r^2 + (v\,t)^2$$

$$r^2 + 2\,h\,r + h^2 = r^2 + (v\,t)^2$$

$$2\,h\,r + h^2 = v^2\,t^2$$

Limit $h \longrightarrow 0$ (neglect $O(h^2)$):

$$2\,h\,r = v^2\,t^2$$

$$h = \frac{1}{2}\,\frac{v^2}{r}\,t^2 = \frac{1}{2}\,a\,t^2$$

$$a = \frac{v^2}{r}$$

Position and Velocity Vectors: Tipler-Mosca figure 3-25.

The angle $\triangle\theta$ between $\vec{v}_1$ and $\vec{v}_2$ is the same as that between $\vec{r}_1$ and $\vec{r}_2$, because the position and velocity vectors must remain mutually perpendicular. The magnitude of the acceleration can be found from the following relations, which hold for in the limit $\triangle\theta \rightarrow 0$, i.e. for very small angles $\triangle\theta$.

$$\triangle\theta = \frac{\triangle r}{r} = \frac{\triangle v}{v}$$

$$\triangle v = \triangle\theta\, v = \triangle r\, \frac{v}{r}$$

$$\triangle r = v\, \triangle t$$

$$\triangle t = \frac{\triangle r}{v}$$

Therefore,

$$a = \lim_{\triangle t \to 0} \frac{\triangle v}{\triangle t} = \lim_{\triangle t \to 0} \frac{\triangle r}{\triangle r} \frac{v^2}{r} = \frac{v^2}{r}$$

$$a = \frac{v^2}{r}$$

The Period T

The time T required for one complete revolution is called the period. For constant speed

$$v = \frac{2\pi\, r}{T} \ \text{holds.}$$

Components of acceleration for a particle moving along an arbitrary curve with varying speed: Figure 3-26 of Tipler-Mosca.

We can treat a portion of the curve as an arc of a circle.

The has then a component of acceleration tangent to the circle

$$\frac{dv}{dt}$$

as well as a radially inward centripetal acceleration

$$\frac{v^2}{r}$$

Centripetal Force:

As with any acceleration, there must be a force in the direction of the acceleration. For centripetal accelerations it is called the centripetal force

$$\vec{F}_{cp} = -m\frac{v^2}{r}\hat{r}$$

where $\hat{r}$ is the unit vector in the direction of $\vec{r}$:

$$\hat{r} = \frac{\vec{r}}{r}$$

Centrifugal Force:

This is the force opposite to the centripetal force, which acts on the entity, which pulls the object towards the center of the circle.

$$\vec{F}_{cf} = +m\frac{v^2}{r}\hat{r} = -\vec{F}_{cp}$$

Example: bucket of water in vertical, circular motion of constant speed.

The force of the water onto the bottom of the bucket is:

$$F_{top} = m\,\frac{v^2}{r} - m\,g$$

at the top of the circle, and

$$F_{bot} = m\,\frac{v^2}{r} + m\,g$$

at the bottom of the circle.

How fast must velocity be, such that the water does not spill?

$$1.\;\; v > \sqrt{g/r} \qquad\qquad 2.\;\; v > \sqrt{g\,r}$$