Announcement

1. Dr. Baba Jain will be holding a help session on Friday, Oct. 10, at 3:30 pm in UPL 110.
2. Look up your mini scores on the CAPA system and compare with your records!
3. Dr. Frawley is responsible for mini 3. Results were not good (7.9 average).
4. Mini 4: Next Thursday. I will spend 15-20 minutes today on preparation.

Rotation (Chapter 9 of Tipler-Mosca)

Figure 9-2 of Tipler-Mosca shows a disk, which rotates about a fixed axis perpendicular to the disk and through its center.

Rigid body: As the disk turns, the distance between any two particles that make up the disk remains fixed.

Consider a particle at distance r from the center and let θ be the angle measures counterclockwise from a fixed reference line. When the disk rotates through and angular displacement $d\theta$, measured in radians (rad), the particle moves through a circular arc of length

$$ds = r\,|d\theta|\ .$$

The time rate of change of the angle is the angular velocity

$$\omega = \frac{d\theta}{dt} \ .$$

Question: A CD-ROM disk is rotating at 3000 revolutions per minute. What is the angular speed in radians per second?

1. 284 rad/s 2. 314 rad/s 3. 334 rad/s.

Angular acceleration:

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} \ .$$

Tangential velocity:

$$v_t = \frac{ds}{dt} = r\,\frac{d\theta}{dt} = r\,\omega \ .$$

Tangential acceleration:

$$a_t = \frac{dv_t}{dt} = r\,\frac{d\omega}{dt} = r\,\alpha \ .$$

Do not confuse this with the centripetal acceleration

$$a_c = \frac{v_t^2}{r} = \frac{(r\omega)^2}{r} = r\omega^2 \ .$$

Rotational Kinetic Energy

$$K_{\mathrm{rot}} = \frac{1}{2}\sum_i m_i\, v_i^2 = \frac{1}{2}\sum_i m_i\, (r_i\, \omega)^2 = \frac{1}{2}\left(\sum_i m_i\, r_i^2\right)\, \omega^2$$

The quantity $\qquad I = \sum_i m_i\, r_i^2 \ ,$

where r_i is the distance of particle i from the rotation axis, is called Moment of Inertia. The kinetic energy becomes

$$K_{\mathrm{rot}} = \frac{1}{2} I\, \omega^2 \ .$$

For a continuous object:

$$I = \int r^2\, dm \ .$$

Calculating the Moment of Inertia

Discrete Systems of Particles: Simply apply the definition

$$I = \sum_i m_i \, r_i^2 \ .$$

Examples (PRS):

For figure 9-3 of Tipler-Mosca the result is

$$I = n \, m \, a^2$$

where n is an integer. Push this number.

For figure 9-4 of Tipler-Mosca n is another (or the same?) integer. Push this number.

Continuous Objects:

The moment of inertia is calculated by integration. Examples folow.

1. Uniform stick about an axis perpendicular to the stick and through one end (figure 9-5 of Tipler-Mosca).

$$I = \int_0^L x^2 \, dm = \int_0^L x^2 \, \frac{M}{L} \, dx =$$

pick one!

$$1. \;\; \frac{1}{3} M \, L^3 \qquad 2. \;\; \frac{1}{3} M \, L^2 \qquad 3. \;\; \frac{1}{2} M \, L^2 \; .$$

2. Hoop of radius R about a perpendicular axis through its center (figure 9-6 of Tipler-Mosca).

$$I = \int r^2\, dm = R^2 \int dm = M\, R^2 \ .$$

3. Uniform disk of radius R about a perpendicular axis through its center (figure 9-7 Tipler-Mosca).

$$dm = \frac{M}{A}\, dA_r \quad \text{where} \quad A = \pi\, R^2 \quad \text{and} \quad A_r = \pi\, r^2$$

so that

$$\int_0^R dA_r = \int_0^R 2\, \pi\, r\, dr = \pi\, r^2\Big|_0^R = \pi\, R^2 = A \ .$$

Therefore,

$$dm = \frac{M}{\pi R^2} 2 \pi r \, dr = \frac{2M}{R^2} r \, dr$$

and

$$I = \int r^2 \, dm = \frac{2M}{R^2} \int_0^R r^3 \, dr =$$

pick one!

$$1. \ \frac{1}{4} M R^4 \qquad 2. \ \frac{1}{2} M R^4 \qquad 3. \ \frac{1}{2} M R^2 \qquad 4. \ \frac{2}{3} M R^2 \ .$$

3. Uniform cylinder of radius R about its axis (figure 9-8 of Tipler-Mosca). Using the result for the uniform disk, we have

$$I = \frac{1}{2}\,R^2 \int dm_{\rm disk} = \frac{1}{2}\,M\,R^2 \ .$$

Results for Uniform Bodies of Various Shapes are collected in Table 9-1 of Tipler-Mosca!

Examples we had:

$$\text{hoop} \quad I = M\,R^2 \ , \qquad \text{full disk} \quad I = \frac{1}{2}\,M\,R^2 \ .$$

Rolling Objects without Slipping

The point of contact moves a distance

$$s = R\,\phi\ .$$

The velocity and acceleration of the CM are

$$v_{\rm cm} = R\,\omega \quad \text{and} \quad a_{\rm cm} = R\,\alpha\ .$$

The kinetic energy of a rolling object is

$$K = \frac{1}{2}\,M\,v_{\rm cm}^2 + \frac{1}{2}\,I\,\omega^2\ .$$

Example: Objects rolling down an inclined plane. The terminal velocity follows from energy conservation:

$$m\,g\,h = K = m\,\frac{v_{\rm cm}^2}{2} + I\,\frac{v_{\rm cm}^2}{2\,R^2} = \left(m + \frac{I}{R^2}\right)\,\frac{v_{\rm cm}^2}{2}$$

$$v_{\rm cm}^2 = \frac{2\,m\,g\,h}{m + I/R^2}$$

where the moment of inertia I is about the CM axis.

A hoop (1), a full disk (2), and a body on tiny wheels (3) have equal masses. Starting from rest, they roll down the same inclined plane. In which order do they arrive?

1. 123   2. 132   3. 213   4. 231   5. 312   6. 321.

Parallel Axis Theorem

The moments of inertia about an axis through the CM and a parallel axis a distance h away (figure 9-14 of Tipler) are related by

$$I_h = I_{\rm cm} + M\,h^2 \ .$$

This theorem simplifies often the calculation of the moment of inertia, because it allows to pick an axis for which the calculation is particularly simple.