Static Equilibrium (Chapter 12)

1. The net external force acting on the body must remain zero:

$$\sum \vec{F} = 0 \ .$$

2. The next external torque about any point must remain zero:

$$\sum \vec{\tau} = 0 \ .$$

Conveniently, the net torque due to gravity about any point O can be calculated as if the entire weight $\vec{W}$ were applied at a the center of gravity

$$\vec{\tau}_{\rm net} = \vec{r}_{\rm cg} \times \vec{W} \ .$$

In a uniform gravitational field: $\vec{r}_{\rm cg} = \vec{r}_{\rm cm}$.

Balanced Bar

Assume, the is one pivot. The condition is

$$\sum \vec{\tau} = 0$$

for the torques.

Stability of Rotational Equilibrium

Figures 12-16 and 12-17 of Tipler-Mosca: Cone, rod.

Sign hanging from a rod

Figures 12-6 and 12-7 of Tipler-Mosca. The weight of the sign and other numbers are given in the book.

Problem: Find the tension T in the wire and the force $-\vec{F}$ of the rod against the wall.

1. Draw a free body diagram for the rod (figure 12-7 of Tipler-Mosca).

2. $\sum \tau = 0$ about point O:

$$T\,L\,\sin\theta - M\,g\,L - m\,g\,\frac{L}{2} = 0\ .$$

3. Use trigonometry to solve for θ:

$$\theta = \tan^{-1}\left(\frac{1}{2}\right) = 26.6^\circ \ .$$

4. Solve for T:

$$T = \frac{M\,g}{\sin\theta} + \frac{m\,g}{2\,\sin\theta} = 483\,\mathrm{N} \ .$$

5.

$$0 = \sum F_x = F_x + T_x \quad \text{and} \quad 0 = \sum F_y = F_y + T_y - M\,g - m\,g$$

$$\begin{aligned} F_x = -T_x &= +T\,\cos(\theta) = 432\,\mathrm{N} \\ F_y = -T_y + M\,g + m\,g &= +T\,\sin(\theta) + M\,g + m\,g = 19.2\,\mathrm{N} \ . \end{aligned}$$

Ladder

Figures 12-10 and 12-11 of Tipler-Mosca. The weight of the ladder and other numbers are given in the figures.

Question: What is the minimum coefficient of static friction μ_s, so that the ladder does not slip.

1. Draw a free body diagram (figure 12-11 of Tipler-Mosca).

2.

$$f_s = \mu_s F_n$$

3.

$$0 = \sum F_x = f_s - F_1 \quad \text{and} \quad 0 = \sum F_y = F_n - w$$

4. Solve for f_s and F_n:

$$f_s = F_1 \quad \text{and} \quad F_n = w = 60\,\mathrm{N} \ .$$

5. $\sum \tau = 0$ about an axis directed out of the page and through the foot of the ladder:

$$4\,\mathrm{m}\,F_1 - 1.5\,\mathrm{m}\,w = 0 \ .$$

6. Solve for f_s:

$$f_s = F_1 = \frac{1.5\,\mathrm{m}\,w}{4\,\mathrm{m}} = 22.5\,\mathrm{N} \ .$$

7.

$$\mu_s = \frac{f_s}{F_n} = \frac{22.5\,\mathrm{N}}{60\,\mathrm{N}} = 0.375$$

Stress and Strain

When a solid object is subjected to forces that tend to stretch, shear, or compress it, its shape changes.

If the object returns to its original shape when the forces are removed, it is said to be elastic.

Most objects are elastic for forces up to a certain limit, called the elastic limit.

A force streching an object is called a tensile force (figure 12-20 of Tipler-Mosca).

The fractional change in the length $\frac{\triangle L}{L}$ is called the strain:

$$\text{Strain} = \frac{\triangle L}{L} \; .$$

The ratio of the force F to the cross-sectional area A is called the tensile stress:

$$\text{Stress} = \frac{F}{A} \; .$$

Figure 12-21 of Tipler-Mosca shows a graph of stress versus strain for a typical solid bar. The ratio of stress to strain in the linear region (Hooke's law applies there) of the graph is a constant called Young's modulus Υ:

$$\Upsilon = \frac{\text{stress}}{\text{strain}} = \frac{F/A}{\triangle L/L} \; .$$

The stress at which breakage occurs is called tensile strength.

If a bar is subjected to forces that compress it rather than stretch it, the notation is: compressive stress instead of stress, and compressive strength for the breakoint force.

Similar treatment of shear forces: Figure 12-22 of Tipler-Mosca.

Fluids (Chapter 13)

Density:

$$\rho = \text{Density} = \frac{\text{mass}}{\text{volume}} = \frac{m}{V} \ .$$

Liter (L): An often used unit for the volume of fluids:

$$1\,\text{L} = 10^3\,\text{cm}^3 = 10^{-3}\,\text{m}^3 \ .$$

Because the gramm was originally defined as the mass of one cubic centimeter of water, the weight of 1 L water at 4° C is 1.00 kg.

The ratio of the density of a substance to that of water is called its specific gravity.

Pressure

When a body is submerged in a fluid, the fluid exerts a force perpendicular to the surface of the body at each point of the surface. This force per unit area is called pressure P of the fluid:

$$P = \frac{F}{A} \ .$$

The SI unit for pressure is Pascal (Pa):

$$1\,\mathrm{Pa} = 1\,\mathrm{N/m^2} \ .$$

In the U.S. pressure is usually given in pounds per square inch $(\mathrm{lb/in.^2})$.

Another common unit is the atmosphere (atm), which equals approximately the air pressure at sea level. One atmosphere is now defined in kilopascals:

$$1\,\mathrm{atm} = 101.325\,\mathrm{kPa} \approx 14.70\,\mathrm{lb/in.^2} \ .$$

This is quite a large pressure on you. Do you believe it?

Pressure tends to compress an object. The ratio of the increase in the pressure $\triangle P$ to the fractional decrease in volume $(-\triangle V/V)$ is called bulk modulus:

$$B = -\frac{\triangle P}{\triangle V/V} \; .$$

The compressibility is the reciprocal of the bulk modulus.

Liquids and solids are relatively incompressible. They have large values of B, relatively independent of temperature and pressure.

Gases are easily compressed, and their values for B depend strongly on temperature and pressure.

The weight of an incompressible liquid in a column of cross-sectional area A and height $\triangle h$ is

$$w = m\, g = (\rho\, V)\, g = \rho\, A\, \triangle h\, g \quad (\rho \text{ constant}) \; .$$

If P_0 is the pressure at the top and P is the pressure at the bottom, we have

$$P\, A - P_0\, A = \rho\, A\, \triangle h\, g$$

or

$$P - P_0 = \rho \triangle h \, g \ .$$

The pressure depends only on the depth of the water.

Pascal's principle (Blaire Pascal, 1623–1662):

- A pressure change applied to an enclosed liquid is transmitted undiminisched to every point in the liquid and to the walls of the container.

Example: Hydraulic lift (Figure 13-4 of Tipler-Mosca).

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

PRS: Assume the large piston has a radius or 10 cm and the small piston a radius of 1 cm. What force must be applied to the small piston, to raise a car of $15,000$ N weight?

1. $1,500\,\mathrm{N}$ 2. $150\,\mathrm{N}$ 3. $15\,\mathrm{N}$.

Archimedes' Principle

The force exerted by a fluid on a body wholly or partially submerged in it is called the buoyant force.

- A body wholly or partially submerged in a fluid is buoyed up by a force equal to the weight of the displaced fluid.

Example: Figure 13-9 amd 13-10 of Tipler-Mosca.

Application: Density Measurements.