Waves Encountering Barriers

Reflection and Refraction:

When a wave is incident on a boundary that separates two regions of different wave speed, part of the wave is reflected and part is transmitted. Figure 15-17 of Tipler-Mosca shows: (a) A pulse on a light string that is attached to a heavier string. (c) A pulse on a heavy string that is attached to a light string. In the first case the replected pulse is inverted in the second not. The same happens with a light and a heavy spring attached to one another.

In three dimensions, a boundary between two regions of differing wave speed is a surface. Figure 15-19 of Tipler-Mosca shows a ray incident on such a surface.

The reflected ray makes an angle with the normal to the surface equal to that of the incident ray.

The transmitted ray is bent towards or away from the normal, a process called refraction. When the wave speed in the second medium is greater than that in the incident medium, the ray is bent away from the normal. As the angle of incidence is increased, the angle of refraction increases unit it reaches 90^0. This defines a critical angle of incidence and for even greater incident angles, there is no refracted ray, a phenomenon called total internal reflection.

In total internal reflection, the wave function drops to zero exponentially fast, so that it becomes negligible within a few wavelength from the surface. This can lead to barrier penetration or tunneling.

Diffraction:

A wave encountering a small obstacle tends to bend around the obstacle. This bending of the wavefront is called diffraction.

When a wave encounters a barrier with an aperture, which is much smaller than the wavelength, the wave bends and spreads out as a spherical circular wave. This distinguishes the waves from particles (Figure 15-22 of Tipler-Mosca), which can be regarded as waves with a wavelength so small that any aperture will be large in comparison. In the ray approximation waves are traveling with no diffraction.

Sound waves with frequencies above 20,000 Hz are called ultrasonic waves. Because of their small wavelength they can be send out to be reflected from small objects. Used by bats, sonar (sound navigation and ranging), sonogram for diagnostic purposes.

The Doppler Effect

When a wave source and a receiver are moving relative to each other, the frequencey is not the same as that omitted. This is called the Doppler effect after the Austrian physicist who predicted this phenomenon.

The change in the frequency is slightly different depending on whether the source or the receiver moves relative to the medium. When the source moves, the wavelength changes, and the new frequency if found from the relation $f = v/\lambda$. When the receiver moves, the frequency changes, while the wavelength is unchanged.

Consider that a source moves with speed u_s relative to the medium. The frequency of the source is f_0. The waves in front of the source are compressed, whereas behind the source they are farther appart (Figure 15-24 of Tipler-Mosca). When v is the speed of the wave in the medium, the new wavelength becomes

$$\lambda' = \frac{v \pm u_s}{f_0} \ .$$

In front of the source the minus sign applies, behind the source the plus sign. The receiver (at rest) gets then the frequency

$$f' = \frac{v}{\lambda'} = \frac{v\, f_0}{v \pm u_s} = \frac{f_0}{1 \pm u_s/v} \ .$$

If the receiver moves with velocity u_r relative to the medium, and the source is at rest, the number of waves crests that pass by per second becomes

$$f' = \frac{v \pm u_r}{\lambda_0} = \left(1 \pm \frac{u_r}{v}\right)\, f_0 \ .$$

Here the plus sign stand when the receiver moves towards the source, otherwise the minus sign applies.

When receiver and source move, one combines the two equations in the obvious way.

Example: The radar used by police to catch speeders relies on the Doppler effect. Electromagnetic waves emitted by the transmitter are reflected by the car and the Doppler shift is measured.

Schock Waves and Mach Number:

If a source moves with speed greater than the wave speed v, there will be no waves in front of the source.

When a source accelerates to approach (and pass) wave speed, the waves pile up as a shock wave. In case of a sound wave, this is heard as a sonic boom.

When the source travels at a constant speed $u > v$, the wave is confined to a cone of angle

$$\sin(\theta) = \frac{v\,t}{u\,t} = \frac{v}{u} = \text{Mach number},$$

see Figure 15-24 of Tipler-Mosca.

Superpositions of Waves (Chapter 16)

When two waves meet in space, they add algebraically (superposition). The superposition of harmonic waves is called interference. In 1801 Young observed the interference of light. Davisson and Germer observed in 1927 the interference of electron waves.

The principle of superposition:

When two or more waves combine, the resultant wave is the algebraic sum of the individual waves:

$$y_3(x,t) = y_1(x,t) + y_2(x,t) \ .$$

Examples: Figure 16-1 of Tipler-Mosca.

Interference of Harmonic Waves:

Two wave sources that are in phase or have a constant phase difference are said to be coherent, otherwise they are said to be incoherent.

We consider the superposition of two (Figure 16-2 Tipler-Mosca) coherent waves

$$\begin{aligned} y_1 &= y_0 \sin(k\,x - \omega\,t) \\ y_2 &= y_0 \sin(k\,x - \omega\,t + \delta) \\ y_3 = y_1 + y_2 &= y_0 \sin(k\,x - \omega\,t) + y_0 \sin(k\,x - \omega\,t + \delta)\ . \end{aligned}$$

Using the trigonometric identity

$$\sin(\theta_1) + \sin(\theta_2) = 2\,\cos[(\theta_1 - \theta_2)/2]\,\sin[(\theta_1 + \theta_2)/2]$$

$$y_3 = 2y_0\,\cos(\delta/2)\,\sin(k\,x - \omega\,t + \delta/2)\ .$$

The resulting wave has interesting properties:

If the two waves are in phase, $\delta = 0$, the amplitude of y_3 is $2y_0$, constructive interference (Figure 16-3 of Tipler-Mosca).

If the two wave are 180^o out of phase, $\delta = \pi$, then $y_3 = 0$, destructive interference (Figure 16-4 of Tipler-Mosca).

Beats

This phenomenon is caused by the interference of sound waves with slightly different frequencies.What do we hear? For equal amplitudes we have at a fixed point, up to a phase constant, the pressure fluctuation

$$p = p_1 + p_2 = p_0 \sin(\omega_1 t) + p_0 \sin(\omega_2 t)$$

$$\begin{aligned} p &= 2p_0 \cos[(\omega_1 - \omega_2)\, t/2] \sin[(\omega_1 + \omega_2)\, t/2] \\ &= 2p_0 \cos[(\triangle\omega/2)\, t] \sin[(\omega_{\rm av}\, t] \end{aligned}$$

where $\triangle\omega = \omega_1 - \omega_2$ and $\omega_{\rm av} = (\omega_1 + \omega_2)/2$. The frequencies of the factors are

$$f_{\rm beat} = 2\triangle f = \frac{2\triangle\omega}{2\pi} \quad \text{and} \quad f_{\rm av} = \frac{2\omega_{\rm av}}{2\pi} \ .$$

The tone we hear has the average frequency $f_{\rm av}$, whose amplitude $2p_0\, cos(2\pi\, f_{\rm beat}\, t)$ is modulated by the beat frequency, which is much smaller than the average frequency (Figure 16-5 of Tipler-Mosca). Beats can be used to tune a piano.

Phase Difference due to Path Difference

The wave function from two coherent sources, oscillating in phase, can be written as (Figure 16-6 of Tipler-Mosca)

$$p = p_1 + p_2 = p_0 \sin(k\,x_1 + \omega\,t) + p_0 \sin(k\,x_2 + \omega\,t) \ .$$

An example is given in Figure 16-8 of Tipler-Mosca. The phase difference for these two wave function is

$$\delta = k\,(x_2 - x_1) = 2\pi\,\frac{\triangle x}{\lambda} \ .$$

The amplitude is $2p_0 \cos(\delta/2)$ and Figure 16-9 of Tipler-Mosca shows how the intensity varies with the path difference.

The Double-Slit Experiment:

Interference of light is difficult to observe, because a light beam is usually the result of millions of atoms radiating incoherently. Coherence in optics is commonly achieved by splitting the light beam from a single source. One method of achieving this is by diffraction of a light beam by two slits in a barrier (Thomas Young 1801).

Standing Waves (Chapter 16-2)

When waves are confined in space, reflections at both ends cause the wave to travel in both directions. For a string or pipe, there are certain frequencies for which superposition results in a stationary pattern called standing wave. The frequencies that produce these patterns are called resoncance frequencies. Each such frequency with its accompanying wave function is called a mode of vibration. The lowest frequency produces the fundamental mode or first harmonic. For each frequency there are certain points on the string that do not move. Such points are called nodes. Midway between each pair of nodes is a point of maximum amplitude of vibration called an antinode.

String fixed at both ends (Figure 16-10 of Tipler-Mosca):

The standing wave condition is

$$L = n\,\frac{\lambda_n}{2}, \quad n = 1, 2, 3, \ldots$$

and with $f_n \, \lambda_n = v$ the resonance frequencies become

$$f_n = n \, \frac{v}{2L} = n \, f_1, \quad n = 1, 2, 3, \ldots$$

where f_1 is the fundamental frequency. Example: pianos.

String fixed at one end (Figure 16-16 of Tipler-Mosca):

The free end is an antinode. The standing wave condition can thus be written

$$L = n \, \frac{\lambda_n}{4}, \quad n = 1, 3, 5, \ldots$$

and with $f_n \, \lambda_n = v$ the resonance frequencies become

$$f_n = n \, \frac{v}{4L} = n \, f_1, \quad n = 1, 3, 5, \ldots$$

where f_1 is the fundamental frequency. Example: the air column in an organ pipe.

Wave Functions for Standing Waves

Standing wave occur due to the superposition of the reflected waves. When a sting vibrates in its nth mode, a point on the string moves with simple harmonic motion. Therefore, the wave function is given by

$$y(x,t) = A_n(x)\,\cos(\omega_n\,t + \delta_n)$$

where ω_n is the angular frequency, δ_n the phase constant, and $A(x)$ the amplitude, which depends on the location on the string. At an instant where the vibration is at its maximum amplitude, the shape of the string is

$$A_n(x) = A_n\,\sin(k_n\,x)$$

where $k_n = 2\pi/\lambda_n$ is the wave number. The wave function for a standing wave in the nth harmonic can thus be written

$$y_n(x,t) = A_n\,\sin(k_n\,x)\,\cos(\omega_n\,t + \delta_n)\ .$$

Superpositions of Standing Waves

In general, a vibrating system does not vibrate in a single harmonic mode. Instead, the motion consists of a mixture of the allowed harmonics and the wave function is a linear combination of the harmonic wave functions:

$$y(x,t) = \sum_n A_n \sin(k_n x) \cos(\omega_n t + \delta_n)$$

where $k_n = 2\pi/\lambda_n$, $\omega_n = 2\pi f_n$, and A_n, δ_n are constants which depend on the initial position and velocity of the string. Interestingly each wave, which fulfills the appropriate boundary conditions (here $y = 0$ at $x = 0$ and $x = L$), can be expanded in this way.

Harmonic Analysis and Synthesis:

Waves can be analyzed in terms of harmonics. Example: Figure 16-24 of Tipler-Mosca shows the relative intensities for a tuning fork, a clarinet, and an oboe, each playing a tone at a fundamental frequency of 440 Hz.
The inverse is harmonics synthesis, the construction of a periodic wave from harmonic components. Example: Figure 16-25 and 16-26 of Tipler-Mosca.

Wave Packets and Dispersion

Pulses, which are not periodic, can also be expanded into sinusoidal waves of different frequencies. However a continuous distributions of frequencies rather than a discrete set of harmonics is needed. These are wave packets. The characteristic feature of a wave pulse is that it has a beginning and an end. If the duration of the pulse is $\triangle t$, the range of frequencies $\triangle\omega$, needed to describe the impulse, is given by the relation

$$\triangle\omega\,\triangle t \sim 1\ .$$

E.g., if $\triangle t$ is very small, $\triangle\omega$ is very large and vice versa.

A wave pulse produced by a source of duration $\triangle t$ has a width $\triangle x = v\,\triangle t$ in space, where v is the wave speed. A range of frequencies $\triangle\omega$ implies a range of wave numbers $\triangle k = \triangle\omega/v$. Therefore, $\triangle\omega\,\triangle t \sim 1$ implies

$$\triangle k\,\triangle x \sim 1\ .$$

If a wave packet is to maintain its shape as it travels, all of the components must travel at the same speed. A medium where this happens is called non-dispersive medium.

Air is a non-dispersive medium for sound waves, but solids and liquids are generally not.

A familiar example for the dispersion of light waves is the rainbow.

When the speed of the wave component depends only slightly on the their wavelength, the wave packet changes shape only slowly as it travels. However, the speed of the wave packet, called group velocity, is not the same as the (average) speed of the components, called phase velocity. For example, the group velocity of surface waves in deep water is half the phase velocity.