Kinematics

Displacement of a point particle:

$$\triangle\vec{x} = \vec{x}_2 - \vec{x}_1$$

where $\vec{x}_1$ is the position at time t_1 and $\vec{x}_2$ is the position at time t_2, $t_2 > t_1$.

Instantaneous velocity

$$\vec{v} = \frac{d\,\vec{x}}{d\,t}$$

This is the slope of the tangent of the curve $\vec{x}(t)$ at t and called derivative.

The instantaneous acceleration is:

$$\vec{a} = \frac{d\,\vec{v}}{d\,t} = \frac{d^2\vec{x}}{dt^2}$$

Motion With Constant Acceleration

$$\frac{d\,\vec{v}}{d\,t} = \vec{a} = \vec{a}_{\rm average}$$

Integration:

$$\vec{v} = \frac{d\,\vec{x}}{d\,t} = \vec{v}_0 + \vec{a}\,t$$

Here $\vec{v}_0$ is the velocity at time zero, the first initial condition.

Second integration:

$$\vec{x} = \vec{x}_0 + \vec{v}_0\,t + \frac{1}{2}\,\vec{a}\,t^2$$

Here $\vec{x}_0$ is the second initial condition, the position at time zero.

SI Units: m and s.

1D:

$$x = x_0 + v_0\,t + \frac{1}{2}\,a\,t^2$$

How long does an object take to fall from a 200 m heigh tower?

1. 20.4 s
2. 40.7 s
3. 6.39 s
4. 4.52 s

What is the velocity of the object when it hits the ground?

1. 200 m/s
2. 100 m/s
3. 62.6 m/s
4. 125.2 m/s

Newton's Laws

1. Law of inertia. An object continues to travel with constant velocity (including zero) unless acted on by an external force.

2. The acceleration $\vec{a}$ of an object is given by

$$m\,\vec{a} = \vec{F}_{\rm net} = \sum_i \vec{F}_i$$

   where m is the mass of the object and $\vec{F}_{\rm net}$ the net external force.

3. Action $=$ Reaction. Forces always occur in equal and opposite pairs. If object A exterts a force on object B, an equal but opposite force is exterted by object B on A.

Friction

An object may not move because the external force is balanced by the force f_s of static friction. Its maximum value $f_{s,max}$ is obtained when any further increase of the external force will cause the object to slide. To a good approximation $f_{s,max}$ is simply proportional to the normal force

$$f_{s,max} = \mu_s \, F_n$$

where μ_s is called the coefficient of static friction. Kinetic friction (also called sliding friction): Once the box slides, a constant force is needed to keep it sliding at constant velocity. The opposing force is the force of kinetic friction. In a good approximation it is also simply proportional to the normal force

$$f_k = \mu_k \, F_n$$

where μ_k is called the coefficient of kinetic friction.

Experimentally it is found that $\mu_k < \mu_s$.

Work and Energy

Motion With Constant Force:

The work W done by a constant Force $\vec{F}$ whose point of application moves through a distance $\triangle\vec{x}$ is defined to be

$$W = F\,\cos(\theta)\,\triangle x$$

where θ is the angle between the vector $\vec{F}$ and the vector $\triangle\vec{x}$.

If $\triangle\vec{x}$ is along the x-axis, then

$$W = F_x\,\triangle x$$

holds. Work is a scalar quantity that is positive if $\triangle x$ and F_x have the same sign and negative otherwise.

The SI unit of work and energy is the joule (J): $1\,\mathrm{J} = 1\,\mathrm{N}\cdot\mathrm{m} = 1\,\mathrm{kg}\,\mathrm{m}^2\,/\,\mathrm{s}^2$.

Work and Kinetic Energy

There is and important theorem, which relates the total work done on a particle to its initial and final speeds. If $\vec{F}$ is the net force acting on a particle, Newton's second law gives

$$\vec{F} = m\,\vec{a}$$

The total work becomes

$$W_{tot} = m\,\vec{a}\,\triangle\vec{x} = \frac{1}{2}\,m\,\vec{v}_f^2 - \frac{1}{2}\,m\,\vec{v}_i^2$$

The kinetic energy of the particle is defined by:

$$K = \frac{1}{2}\,m\,\vec{v}^2$$

and the mechanical work-kinetic energy theorem states: The total work done on the particle is equal to the change in kinetic energy

$$W_{tot} = K_f - K_i$$

Potential Energy

Often work done by external forces on a system does not increase the kinetic energy of the system, but is instead stored as potential energy.

Conservative Forces:

A force is called conservative when its total work done on a particle along a closed path is zero (figure 6-22 of Tipler).

Potential-Energy Function:

For conservative forces a potential energy function U can be defined, because the work done between two positions 1 and 2 does not depend on the path:

$$\triangle U = U_2 - U_1 = -\int_{s_1}^{s_2} \vec{F} \cdot d\vec{s}$$

$$dU = -\vec{F} \cdot d\vec{s} \ \text{ for infinitesimal displacements.}$$

Example: Gravitational potential energy near the earth's surface.

$$dU = -\vec{F} \cdot d\vec{s} = -(-m\,g\,\hat{j}) \cdot (dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}) = m\,g\,dy$$

$$U = \int dU = m\,g \int_{y_0}^{y} dy' = m\,g\,y - m\,g\,y_0$$

Work-Energy Theorem with Kinetic Friction

Non-conservative Forces: Not all forces are conservative. Friction is an example of a non-conservative force. The energy dissipated by friction is thermal energy (heat):

$$f \triangle s = \triangle E_{\mathrm{therm}}$$

where f is the frictional force applied along the distance $\triangle s$. The work-energy theorem reads then

$$W_{\mathrm{ext}} = \triangle E_{\mathrm{mech}} + \triangle E_{\mathrm{therm}} \ .$$

Momentum Conservation

The Center of Mass (CM):

The CM $\vec{r}_{\rm cm}$ moves as if all the external foces acting on the system were acting on the total mass M of the system located at this point. In particular, it moves with constant velocity, if the external forces acting on the system add to zero.

Definition:

$$M\,\vec{r}_{\rm cm} = \sum_{i=1}^{n} m_i\,\vec{r}_i \quad \text{where} \quad M = \sum_{i=1}^{n} m_i \ .$$

Here the sum is over the particles of the system, m_i are the masses and $\vec{r}_i$ are the position vectors of the particles. In case of a continuous object, this becomes

$$M\,\vec{r}_{\rm cm} = \int \vec{r}\,dm$$

where dm is the position element of mass located at position $\vec{r}$.

Momentum:

The mass of a particle times its velocity is called momentum

$$\vec{p} = m\,\vec{v}\ .$$

Newton's second law can be written as

$$\vec{F}_{\rm net} = \frac{d\vec{p}}{dt} = \frac{d(m\vec{v})}{dt} = m\,\frac{d\vec{v}}{dt} = m\,\vec{a}$$

as the masses of our particles have been constant.

The total momentum $\vec{P}$ of a system is the sum of the momenta of the individual particles:

$$\vec{P} = \sum_{i=1}^{n} \vec{p}_i = \sum_{i=1}^{n} m_i\,\vec{v}_i = M\,\vec{v}_{\rm cm}$$

Differentiating this equation with respect to time, we obtain

$$\frac{d\vec{P}}{dt} = M\,\frac{d\vec{v}_{\rm cm}}{dt} = M\,\vec{a}_{\rm cm} = \vec{F}_{\rm net,ext}$$

The law of momentum conservation: When the net external force is zero, the total momentum is constant

$$\vec{F}_{\rm net,ext} = 0 \quad \Rightarrow \quad \vec{P} = \text{constant}.$$

Example:

Inelastic scattering, figure 8-30 of Tipler-Mosca.

A bullet of mass m_1 is fired into a hanging target of mass m_2, which is at rest. The bullet gets stuck in the target. Find the speed v_i of the bullet from the joint velocity v_f of bullet and target after the collision.

PRS:

The result is

$$1. \;\; v_i = \frac{m_1 + m_2}{m_1} v_f \qquad 2. \;\; v_i = \frac{m_1 + m_2}{m_2} v_f$$

Solution: Momentum conservation gives

$$p_i = m_1\, v_i = (m_1 + m_2)\, v_f = p_f$$

Given the initial speed v_i of the bullet, what is the speed of the combined systems after the inelastic scattering?

$$1. \quad v_f = \frac{m_1\, v_i}{m_1 + m_2} \qquad\qquad 2. \quad v_f = \frac{(m_1 + m_2)\, v_i}{m_2} \qquad\qquad .$$

How high will the combined system swing?

$$1. \quad h = \frac{v_f}{\sqrt{2\, g}} \qquad\qquad 2. \quad h = \frac{v_f^2}{2\, g} \qquad\qquad .$$

Rotation

1. The angular velocity $\vec{\omega}$. Direction: Right-hand-rule.

2. In accordance with the right-hand-rule the torque is defined as a vector: $\vec{\tau} = \vec{r} \times \vec{F}$.

3. Angular Momentum Definition: $\vec{L} = \vec{r} \times \vec{p}$.

   Like the torque angular momentum is defined with respect to the point in space where the position vector $\vec{r}$ originates. For a rotation around a symmetry axis we find $\vec{L} = I\,\vec{\omega}$ (magnitude $L = I\,\omega$).

4. Rotational kinetic energy: $K_{\text{rot}} = \frac{1}{2}\,I\,\omega^2$.

Torque and Angular Momentum

The net external torque acting on a system equals the rate of change of the angular momentum of the system:

$$\sum_i \vec{\tau}_{i,\mathrm{ext}} = \frac{d\vec{L}}{dt}$$

Conservation of Angular Momentum

If the net external torque acting on a system is zero, the total angular momentum of the system is constant.

$$\frac{d\vec{L}}{dt} = \vec{\tau}_{\mathrm{net}} = 0 \quad \Rightarrow \quad \vec{L} = \mathrm{constant}.$$

Gravity

Gravitational foce:

$$|F_{12}| = G\,\frac{m_1\,m_2}{(r_{12})^2}$$

Fluids (Chapter 13)

Density:

$$\rho = \text{Density} = \frac{\text{mass}}{\text{volume}} = \frac{m}{V} \; .$$

Liter (L): An often used unit for the volume of fluids:

$$1\,\text{L} = 10^3\,\text{cm}^3 = 10^{-3}\,\text{m}^3 \; .$$

Because the gramm was originally defined as the mass of one cubic centimeter of water, the weight of 1 L water at 4° C is 1.00 kg.

Pressure:

When a body is submerged in a fluid, the fluid exerts a force perpendicular to the surface of the body at each point of the surface. This force per unit area is called pressure P of the fluid:

$$P = \frac{F}{A} \; .$$

The SI unit for pressure is Pascal (Pa):

$$1\,\mathrm{Pa} = 1\,\mathrm{N/m^2}\ .$$

Another common unit is the atmosphere (atm), which equals approximately the air pressure at sea level. One atmosphere is now defined in kilopascals:

$$1\,\mathrm{atm} = 101.325\,\mathrm{kPa} \approx 14.70\,\mathrm{lb/in.^2}\ .$$

The weight of an incompressible liquid in a column of cross-sectional area A and height $\triangle h$ is

$$w = m\,g = (\rho\,V)\,g = \rho\,A\,\triangle h\,g \quad (\rho\ \text{constant})\ .$$

If P_0 is the pressure at the top and P is the pressure at the bottom, we have

$$P\,A - P_0\,A = \rho\,A\,\triangle h\,g$$

or

$$P - P_0 = \rho\,\triangle h\,g\ .$$

The pressure depends only on the depth of the water.

Pascal's principle (Blaire Pascal, 1623–1662):

- A pressure change applied to an enclosed liquid is transmitted undiminisched to every point in the liquid and to the walls of the container.

Archimedes' Principle

The force exerted by a fluid on a body wholly or partially submerged in it is called the buoyant force.

- A body wholly or partially submerged in a fluid is buoyed up by a force equal to the weight of the displaced fluid.

Fluids in Motion

The general behavior of fluid in motion is very comples, because of the phenomen of turbulence. But there are some easy concepts governing the non-turbulent, steady-state flow of an incompressible fluid.

Continuity equation (Figure 13-13 of Tipler-Mosca):

Let v the velocity of the flow and A be the cross-sectional area, the

$$I_v = A\,v = \text{constant} \ .$$

PRS: Assume a velocity of 10 m/s. How many cubic meters/second come out of a pipe of diameter 1 cm ?

1. $3.14\times 10^{-3}\,\mathrm{m}^3/\mathrm{s}$

2. $7.85\times 10^{-4}\,\mathrm{m}^3/\mathrm{s}$

Bernoulli's Equation (Figures 13-14 and 13-15 of Tipler-Mosca):

$$P + \rho\, g\, h + \frac{1}{2}\,\rho\, v^2 = \text{constant}\ .$$

Assume a water tower of height h. At the bottom there is a hole. At which (approximate) speed does the water leak out of the hole?

1. $v = \sqrt{2g\,h}$

2. $v = P$ where P is the pressure at the bottom.