Questions

Definition of the mass: Let $\vec{F}_1$ be the force that acts on object one and $\vec{F}_2$ be the force that acts on object two. The masses of the objects are defined by (pick one):

1.
$$\frac{m_2}{m_1} = \frac{a_1}{a_2} \quad \text{for} \quad \vec{F}_1 \neq \vec{F}_2$$

2.
$$\frac{m_2}{m_1} = \frac{a_1}{a_2} \quad \text{for} \quad \vec{F}_1 = \vec{F}_2$$

String tension: What happens for $\alpha = \beta \to 0$?

1. The tensions approach a finite value.
2. The tensions become larger and larger. Ultimately the string breaks.

Example: Atwood's Machine

Tipler figure 4-50. In the following a masless, frictionless pully and a massless string are assumed.

$$F = (m_1 - m_2)\, g = (m_1 + m_2)\, a$$

$$a = \frac{m_1 - m_2}{m_1 + m_2}\, g$$

Tension in the string:

$$T = m_1\,(g - a) = \left[\frac{m_1\,(m_1 + m_2)}{m_1 + m_2} - \frac{m_1\,(m_1 - m_2)}{m_1 + m_2}\right]$$

$$= \frac{2\, m_1\, m_2\, g}{m_1 + m_2}$$

Friction (Chapter 5-1 of Tipler)

Friction is a complicated phenomenon that arises when the electromagnetic interactions of molecules between two surfaces in close contact lead to a bonding. This bonding is similar to the molecular bonding which keeps objects together.

If an external force acts on a heavy box standing on a floor (see figure 5-1 of Tipler), the box may not move because the external force is balanced by the force f_s of static friction. Its maximum value $f_{s,max}$ is obtained when any further increase of the external force will cause the box to slide. To a good approximation $f_{s,max}$ is simply proportional to the normal force

$$f_{s,max} = \mu_s \, F_n$$

where μ_s is called the coefficient of static friction (some approximate values are given in table 5-1 of Tipler). If the box does not move we have

$$f_s \le f_{s,max} \ .$$

Kinetic friction (also called sliding friction): Once the box slides, a constant force is needed to keep it sliding at constant velocity. The opposing force is the force of kinetic friction. In a good approximation it is also simply proportional to the normal force

$$f_k = \mu_k \, F_n$$

where μ_k is called the coefficient of kinetic friction (some approximate values are also given in table 5-1 of Tipler).

Experimentally it is found that $\mu_k < \mu_s$. Figure 5-3 of Tipler shows the frictional force exerted on the box by the floor as a function of the applied force.

Example: Block on an inclined plane with friction.

Tipler figure 5-6:

$$F_n - m\,g\,\cos(\theta) = m\,a_y = 0$$

$$F_n = m\,g\,\cos(\theta)$$

$$m\,g\,\sin(\theta) - f_s = m\,a_x = 0$$
$$f_s = m\,g\,\sin(\theta)$$

For the maximum:

$$= \mu_s\,F_n = \mu_s\,m\,g\,cos(\theta)$$

$$\mu_s = \frac{\sin(\theta)}{\cos(\theta)} = \tan(\theta)$$

Example: Forces acting on a car.

How to get the car moving: The car has front-wheel drive and is just starting to move, figure 5-19 of Tipler.

The weight is balanced by the normal forces $\vec{F}_n$.

The engine makes the front wheels rotate. What would happen if the road were frictionless?

1. The car would move.
2. The car would not move, the wheels would merely spin.

When the wheel rolls without slipping, the tire thread touching the road is at rest relative to it and the friction between the road and the tire is static friction $f_s \le \mu_s F_n$, figure 5-20 of Tipler.

How to stop the car: The force that brings the car to a stop when it brakes is the force of friction exerted by the road on the tires, figure 5-21 of Tipler. When the frictional force exerted by the road is constant, the acceleration is constant (and negative), and the stopping distance $\triangle x$ is given by

$$0 = v^2 = v_0^2 + 2\, a_x \triangle x$$

$$\triangle x = -\frac{v_0^2}{2\, a_x}$$

Optimal (ideal) acceleration:

$$a_x = -\frac{f_s}{m} = -\frac{\mu_s\, F_n}{m} = -\frac{\mu_s\, m\, g}{m} = -\mu_s\, g$$

Less ideal, when the wheels slip:

$$a_x = -\mu_k\, g$$

In reality one will normally in-between those two values.