Questions

Atwood machine: With

$$m_1 = 251\,g = 0.251\,kg \quad \text{and} \quad m_2 = 250\,g = 0.250\,kg$$

we measured we measured

$$\triangle t = 12\,s \quad \text{for} \quad \triangle x = 1\,m \; .$$

Approximately, this corresponds to the following lower bound on the gravitational constant g (pick one):

1. $5\,m/s^2$
2. $6\,m/s^2$
3. $7\,m/s^2$
4. $8\,m/s^2$
5. $9\,m/s^2$
6. $10\,m/s^2$

Hint: Calculate the accelerataion a from the our measurements of $\triangle t$ and $\triangle x$ and solve

$$(m_1 - m_2)\,g = (m_1 + m_2)\,a$$

for a.

Why is the estimate a lower bound?

1. Because of friction.
2. Because of the mass of the pulley.
3. Because of friction and the mass of the pulley.
4. Because of the tension in the rope.
5. Because of friction, the mass of the pulley and the tension in the rope.

Coefficient of static friction:

We measured

$$\theta_{\max} = 30^0$$

for a block of wood on an inclined plane of wood. This correspond to the coefficient of static friction (use your calculator and pick the result)

$\mu_s =$

1. 0.55
2. $0.55\,m$
3. 0.58
4. $0.58\,m$
5. 0.61
6. $0.61\,m$

Circular Motion (Chapter 5-2 of Tipler)

Centripetal Acceleration:

Pythagorean theorem for figure 5-23 of Tipler:

$$(r + h)^2 = r^2 + (v\,t)^2$$

$$r^2 + 2\,h\,r + h^2 = r^2 + (v\,t)^2$$
$$2\,h\,r + h^2 = v^2\,t^2$$

Limit $h \to 0$ (neglect $O(h^2)$):

$$2\,h\,r = v^2\,t^2$$

$$h = \frac{1}{2}\,\frac{v^2}{r}\,t^2 = \frac{1}{2}\,a\,t^2$$

$$a = \frac{v^2}{r}$$

Position and Velocity Vectors: Tipler figure 5-24.

The angle $\triangle\theta$ between $\vec{v}_1$ and $\vec{v}_2$ is the same as that between $\vec{r}_1$ and $\vec{r}_2$, because the position and velocity vectors must remain mutually perpendicular. The magnitude of the acceleration can be found from the following relations, which hold for in the limit $\triangle\theta \rightarrow 0$, i.e. for very small angles $\triangle\theta$.

$$\triangle\theta = \frac{\triangle r}{r} = \frac{\triangle v}{v}$$

$$\triangle v = \triangle\theta\, v = \triangle r\, \frac{v}{r}$$

$$\triangle r = v\, \triangle t$$

$$\triangle t = \frac{\triangle r}{v}$$

Therefore,

$$a = \lim_{\triangle t \rightarrow 0} \frac{\triangle v}{\triangle t} = \lim_{\triangle t \rightarrow 0} \frac{\triangle r}{\triangle r} \frac{v^2}{r} = \frac{v^2}{r}$$

$$a = \frac{v^2}{r}$$

The Period T:

The time T required for one complete revolution is called the period. For constant speed

$$v = \frac{2\pi\, r}{T} \quad \text{holds.}$$

Components of acceleration for a particle moving along an arbitrary curve with varying speed: Figure 5-25 of Tipler.

We can treat a portion of the curve as an arc of a circle.

The has then a component of acceleration tangent to the circle

$$\frac{dv}{dt}$$

as well as a radially inward centripetal acceleration

$$\frac{v^2}{r}$$

Centripetal Force:

As with any acceleration, there must be a force in the direction of the acceleration. For centripetal accelerations it is called the centripetal force

$$\vec{F}_{cp} = -m\frac{v^2}{r}\hat{r}$$

where $\hat{r}$ is the unit vector in the direction of $\vec{r}$:

$$\hat{r} = \frac{\vec{r}}{r}$$

Centrifugal Force:

This is the force opposite to the centripetal force, which acts on the entity, which pulls the object towards the center of the circle.

$$\vec{F}_{cf} = +m\frac{v^2}{r}\hat{r} = -\vec{F}_{cp}$$

Example: bucket of water in vertical, circular motion.

The force of the water onto the bottom of the bucket is:

$$F_{top} = m\,\frac{v^2}{r} - m\,g$$

at the top of the circle, and

$$F_{bot} = m\,\frac{v^2}{r} + m\,g$$

at the bottom of the circle.

How fast must velocity be, such that the water does not spill?

1. $v > \sqrt{g/r}$
2. $v < \sqrt{g/r}$

Example: Circular Pendulum: Figures 5-28 and 5-29 of Tipler.

$$\vec{T} + \vec{F}_{cf} + m\,\vec{g} = 0$$

$$\vec{T} = T_r\,\hat{r} + T_y\,\hat{y}$$

$$T_r = -T\ \sin(\theta) = -m\,\frac{v^2}{r}$$

$$T_y = T\ \cos(\theta) = m\,g$$

$$\frac{\sin(\theta)}{\cos(\theta)} = \frac{v^2}{g\,r}$$

$$\tan(\theta) = \frac{v^2}{g\,r}$$

$$v = \sqrt{g\,r\ \tan(\theta)}$$

Example: Forces on a car in a banked curve:

Figures 5-32 and 5-33 of Tipler. The optimal angle θ is the one for which the centrifugal force is balanced by the inward component of the normal force. Then:

$$F_n \cos(\theta) - m\, g = 0$$

$$F_n \sin(\theta) - m\, \frac{v^2}{r} = 0$$

$$\tan(\theta) = \frac{v^2}{g\, r}$$