Work and Energy

Motion With Constant Force:

The work W done by a constant Force $\vec{F}$ whose point of application moves through a distance $\triangle\vec{x}$ is defined to be

$$W = F\,\cos(\theta)\,\triangle x$$

where θ is the angle between the vector $\vec{F}$ and the vector $\triangle\vec{x}$, see figure 6-1 of Tipler.

If $\triangle\vec{x}$ is along the x-axis, i.e.

$$\triangle\vec{x} = \triangle x\,\hat{i} = \triangle x\,\hat{x}$$

then

$$W = F_x\,\triangle x$$

holds. Work is a scalar quantity that is positive if $\triangle x$ and F_x have the same sign and negative otherwise.

The SI unit of work and energy is the joule (J)

$$1\,J = 1\,N\cdot m = 1\,kg\,m^2\,/\,s^2$$

Another energy unit frequently used in physics is the electron volt (eV):

$$1\,eV = 1.602\,176\,462\,(63)\times 10^{-19}\,J$$

is the actual value from the National Institute of Standards and Technology (NIST), surfe physics.nist.gov. Often used multiples:

$$meV,\quad keV,\quad MeV\quad \text{and}\quad GeV\,.$$

Which of the following choices corresponds correctly to

$$meV, \quad keV, \quad MeV \quad \text{and} \quad GeV\,?$$

Pick one!

1. $10^3\,eV, \quad 10^4\,eV, \quad 10^6\,eV, \quad 10^9\,eV\,.$
2. $10^{-3}\,eV, \quad 10^2\,eV, \quad 10^3\,eV, \quad 10^6\,eV\,.$
3. $10^{-3}\,eV, \quad 10^3\,eV, \quad 10^6\,eV, \quad 10^9\,eV\,.$
4. $10^{-6}\,eV, \quad 10^3\,eV, \quad 10^6\,eV, \quad 10^9\,eV\,.$
5. $10^{-3}\,eV, \quad 10^2\,eV, \quad 10^3\,eV, \quad 10^9\,eV\,.$
6. $10^{-3}\,eV, \quad 10^2\,eV, \quad 10^3\,eV, \quad 10^6\,eV\,.$

Answer: See table 1-1 of Tipler!

Power

The power P supplied by a force is the rate at which the force does work.

$$P = \frac{dW}{dt}$$

The SI unit of power is called watt (W):

$$1\,W = 1\,J/s$$

$$1\,kW \cdot h = (10^3\,W)\,(3600\,s) = 3.6 \times 10^6\,W \cdot s = 3.6\,MJ$$
$$1\,hp = 505\,ft \cdot lb/s = 746\,W = 0.746\,kW$$

Work and Kinetic Energy

There is and important theorem, which relates the total work done on a particle to its initial and final speeds. If F_x is the net force acting on a particle, Newton's second law gives

$$F_x = m\, a_x$$

and we recall the constant-acceleration formula (Tipler eqn.2-15, or p.3 of my 030107.pdf file) between initial and final speeds:

$$v_f^2 = v_i^2 + 2\, a_x \,\triangle x \ .$$

Now, the total work becomes

$$W_{tot} = m\, a_x \,\triangle x = \frac{1}{2}\, m\, v_f^2 - \frac{1}{2}\, m\, v_i^2$$

where we substituted $a_x \,\triangle x = (v_f^2 - v_i^2)/2$. The kinetic energy of the particle is defined by:

$$K = \frac{1}{2}\, m\, v^2$$

The work-kinetic energy theorem states: The total work done on the particle is equal to the change in kinetic energy

$$W_{tot} = K_f - K_i$$

Work Done by a Variable Force

Tipler figures 6-6 and 6-7:

$$W = \lim_{\triangle x_i \to 0} \sum_i F_x \triangle x_i = \int_{x_1}^{x_2} F_x \, dx$$

$$= \text{area under the } F_x \text{ versus } x \text{ curve.}$$

Example: Work needed to expand a spring from rest.

When we choose $x_0 = 0$ for the rest postion of the spring

$$F_x = k\,(x - x_0) = k\,x$$

Hence,

$$W = \int_0^x F_x \, dx' = \int_0^x k\,x'\,dx' = \frac{1}{2}\,k\,x^2$$