1D Elastic Scattering in the CM Frame

The momentum conservation equations become

$$m_1\, u_{1\mathrm{i}} + m_2,\, u_{2\mathrm{i}} = 0$$

$$m_1\, u_{1\mathrm{f}} + m_2\, u_{2\mathrm{f}} = 0$$

and the equation for the relative velocities in an elastic collision is

$$u_{2\mathrm{f}} - u_{1\mathrm{f}} = u_{1\mathrm{i}} - u_{2\mathrm{i}}\ .$$

Let us calculate $u_{1\mathrm{f}}$:

$$m_1\, u_{1\mathrm{f}} + m_2\, u_{1\mathrm{f}} = -m_2\, u_{1\mathrm{i}} + m_2\, u_{2\mathrm{i}}$$

Now,

$$u_{2\mathrm{i}} = -\frac{m_1}{m_2}\, u_{1\mathrm{i}}$$

Therefore,

$$(m_1 + m_2)\, u_{1\mathrm{f}} = -m_2\, u_{1\mathrm{i}} - m_1\, u_{1\mathrm{i}} = -(m_1 + m_2)\, u_{1\mathrm{i}}$$

Similarly for $u_{2\mathrm{f}}$. For an elastic collision we simply have the equations

$$u_{1\mathrm{f}} = -u_{1\mathrm{i}} \quad \text{and} \quad u_{2\mathrm{f}} = -u_{2\mathrm{i}}$$

in the CM frame.

Example: Figures 8-41 to 8-43 of Tipler $m_1 = 4\,\text{kg}$ and $m_2 = 2\,\text{kg}$. Steps:

1. Find the CM velocity

$$v_{\text{cm}} = \frac{m_1\, v_{1\text{i}} + m_2\, v_{2\text{i}}}{m_1 + m_2} \ .$$

   PRS: Push the (integer) number, which gives v_{cm} in meters per second.

2. Transform the initial velocities to the CM velocities.

   A. PRS: Type in the integer value which gives $|u_{1\text{i}}|$ in meters per second.

   B. PRS: Type in the integer value which gives $|u_{2\text{i}}|$ in meters per second.

3. Solve the collision in the CM frame.

   A. PRS: Type in the integer value which gives $|u_{1\text{f}}|$ in meters per second.

   B. PRS: Type in the integer value which gives $|u_{2\text{f}}|$ in meters per second.

4. Calculate the final velocities in the original frame.

   A. PRS: Type in the integer value which gives $v_{1\mathrm{f}}$ in meters per second.

   B. PRS: Type in the integer value which gives $v_{2\mathrm{f}}$ in meters per second.