Kinematics

Displacement of a point particle:

$$\triangle\vec{x} = \vec{x}_2 - \vec{x}_1$$

where $\vec{x}_1$ is the position at time t_1 and $\vec{x}_2$ is the position at time t_2, $t_2 > t_1$.

Instantaneous velocity

$$\vec{v} = \frac{d\,\vec{x}}{d\,t}$$

This is the slope of the tangent of the curve $\vec{x}(t)$ at t and called derivative.

The instantaneous acceleration is:

$$\vec{a} = \frac{d\,\vec{v}}{d\,t} = \frac{d^2\vec{x}}{dt^2}$$

Motion With Constant Acceleration

$$\frac{d\,\vec{v}}{d\,t} = \vec{a} = \vec{a}_{\text{average}}$$

Integration:

$$\vec{v} = \frac{d\,\vec{x}}{d\,t} = \vec{v}_0 + \vec{a}\,t$$

Here $\vec{v}_0$ is the velocity at time zero, the first initial condition.

Second integration:

$$\vec{x} = \vec{x}_0 + \vec{v}_0\,t + \frac{1}{2}\,\vec{a}\,t^2$$

Here $\vec{x}_0$ is the second initial condition, the position at time zero.

SI Units: m and s.

Exercise: Convert 55 miles per hour into m/s (1 mile = 1.609 km).

Newton's Laws

1. Law of inertia. An object continues to travel with constant velocity (including zero) unless acted on by an external force.

2. The acceleration $\vec{a}$ of an object is given by

$$m\,\vec{a} = \vec{F}_{\rm net} = \sum_i \vec{F}_i$$

   where m is the mass of the object and $\vec{F}_{\rm net}$ the net external force.

3. Action $=$ Reaction. Forces always occur in equal and opposite pairs. If object A exterts a force on object B, an equal but opposite force is exterted by object B on A.

Friction

Friction is a complicated phenomenon that arises when the electromagnetic interactions of molecules between two surfaces in close contact lead to a bonding.

If an external force acts on a heavy box standing on a floor (see figure 5-1 of Tipler), the box may not move because the external force is balanced by the force f_s of static friction. Its maximum value $f_{s,max}$ is obtained when any further increase of the external force will cause the box to slide. To a good approximation $f_{s,max}$ is simply proportional to the normal force

$$f_{s,max} = \mu_s F_n$$

where μ_s is called the coefficient of static friction. If the box does not move we have

$$f_s \le f_{s,max} \ .$$

Kinetic friction (also called sliding friction): Once the box slides, a constant force is needed to keep it sliding at constant velocity. The opposing force is the force of kinetic friction. In a good approximation it is also simply proportional to the normal force

$$f_k = \mu_k F_n$$

where μ_k is called the coefficient of kinetic friction.

Experimentally it is found that $\mu_k < \mu_s$.

Example: A block on an inclined plane with friction:

Figures 5-6 and 5-7 of Tipler.

Work and Energy

Motion With Constant Force:

The work W done by a constant Force $\vec{F}$ whose point of application moves through a distance $\triangle\vec{x}$ is defined to be

$$W = F\,\cos(\theta)\,\triangle x$$

where θ is the angle between the vector $\vec{F}$ and the vector $\triangle\vec{x}$, see figure 6-1 of Tipler.

If $\triangle\vec{x}$ is along the x-axis, i.e.

$$\triangle\vec{x} = \triangle x\,\hat{i} = \triangle x\,\hat{x}$$

then

$$W = F_x\,\triangle x$$

holds. Work is a scalar quantity that is positive if $\triangle x$ and F_x have the same sign and negative otherwise.

The SI unit of work and energy is the joule (J)

$$1\,\mathrm{J} = 1\,\mathrm{N}\cdot\mathrm{m} = 1\,\mathrm{kg}\,\mathrm{m}^2\,/\,\mathrm{s}^2$$

Work and Kinetic Energy

There is and important theorem, which relates the total work done on a particle to its initial and final speeds. If $\vec{F}$ is the net force acting on a particle, Newton's second law gives

$$\vec{F} = m\,\vec{a}$$

The total work becomes

$$W_{tot} = m\,\vec{a}\,\triangle\vec{x} = \frac{1}{2}\,m\,\vec{v}_f^2 - \frac{1}{2}\,m\,\vec{v}_i^2$$

The kinetic energy of the particle is defined by:

$$K = \frac{1}{2}\,m\,\vec{v}^2$$

and the mechanical work-kinetic energy theorem states: The total work done on the particle is equal to the change in kinetic energy

$$W_{tot} = K_f - K_i$$

Potential Energy

Often work done by external forces on a system does not increase the kinetic energy of the system, but is instead stored as potential energy.

Conservative Forces:

A force is called conservative when its total work done on a particle along a closed path is zero (figure 6-22 of Tipler).

Potential-Energy Function:

For conservative forces a potential energy function U can be defined, because the work done between two positions 1 and 2 does not depend on the path:

$$\triangle U = U_2 - U_1 = - \int_{s_1}^{s_2} \vec{F} \cdot d\vec{s}$$

$dU = -\vec{F} \cdot d\vec{s}$ for infinitesimal displacements.

Example: Gravitational potential energy near the earth's surface.

$$dU = -\vec{F}\cdot d\vec{s} = -(-m\,g\,\hat{j})\cdot(dx\,\hat{i}+dy\,\hat{j}+dz\,\hat{k}) = m\,g\,dy$$

$$U = \int dU = m\,g\int_{y_0}^{y} dy' = m\,g\,y - m\,g\,y_0$$

Work-Energy Theorem with Kinetic Friction

Non-conservative Forces: Not all forces are conservative. Friction is an example of a non-conservative force. The energy dissipated by friction is thermal energy (heat):

$$f\,\triangle s = \triangle E_{\rm therm}$$

where f is the frictional force applied along the distance $\triangle s$. The work-energy theorem reads then

$$W_{\rm ext} = \triangle E_{\rm mech} + \triangle E_{\rm therm}\ .$$

Example: Block on an inclined plane with friction, again.

Assume the block (approximated as a point particle) has mass m, starts from rest, and slides down the inclined plane a distance L.

What is its initial potential energy of the block?

The coefficient of kinetic friction is μ_k. How much energy is dissipated in friction, when the block slides down the inclined plane?

Which is the correct equation for the frictional force?

What is the kinetic energy K_b of the block at the bottom of the inclined plane?

What is the velocity of the block at the bottom of the inclined plane?