Wave Motion (Chapter 15)

Waves are moving oscillations. They transport energy and momentum through space without transporting matter. In mechanical waves this happens via a disturbance in a medium.

Transverse waves: The disturbance is perpendicular to the direction of transportation (figure 15-1).

Longitudinal waves: The disturbance is parallel to the propagation (figure 15-2).

Wave Pulses:

In a co-moving reference frame a wave pulse is at all times described by a function $f(x')$. The x coordinate in the Lab system is

$$x = x' \pm v\,t$$

where the wave pulse becomes

$$
\begin{aligned}
y &= f(x - v\,t) \quad \text{wave moving right} \\
\text{or} \quad y &= f(x + v\,t) \quad \text{wave moving left .}
\end{aligned}
$$

The function f is called wave function.

Speed of Waves:

The speed depends on the properties of the medium but is independent of the motion of the source of the waves. For example, the speed of sound from a car depends only on the properties of the air and not on the motion of the car.

For wave pulses on a string one has

$$v = \sqrt{\frac{F}{\mu}}$$

where F is the tension (T is used for the period) and μ the linear mass density.

Harmonic Waves

If one end of a string is attached to a vibrating fork that is moving up and down with an oscillation of frequency f, a sinusoidal wave propagates along the string (figure 15-7 of Tipler). It is called a harmonic wave. The distance after which the wave repeats itself, for example from crest to crest, is the wavelength λ.

As the wave propagates, each point moves up and down in simple harmonic motion, which is perpendicular to the direction of propagation. During one period $T = 1/f$, the wave moves a distance of one wavelength, so its speed is

$$v = \frac{\lambda}{T} = f\,\lambda\ .$$

Later we see that other waves are superpositions of harmonic waves. The wave function of a harmonic wave is

$$y(x,t) = A\,\sin\left[k\,(x - vt) + \delta\right] = A\,\sin(k\,x - \omega\,t + \delta)$$

where A is the amplitude, k the wave number, and for the angular frequency the following equations hold

$$\omega = k\,v = 2\pi\,f = \frac{2\pi}{T} = \frac{2\pi\,v}{\lambda}\ .$$

Exercises:

Assume the string tension is $F = 5.9\,\text{N}$ and the mass per unit length is $\mu = 2.1\,\text{kg/m}$. What is the speed of the wave?

Assume the wavelength is $\lambda = 2\,\text{m}$ and the velocity is the one you just calculated, find the angular frequency.

Find the wave number from the previous data.

Standing Waves (Chapter 16-2)

When waves are confined in space, reflections at both ends cause the wave to travel in both directions. For a string or pipe, there are certain frequencies for which superposition results in a stationary pattern called standing wave. The frequencies that produce these patterns are called resoncance frequencies. Each such frequency with its accompanying wave function is called a mode of vibration. The lowest frequency produces the fundamental mode or first harmonic. For each frequency there are certain points on the string that do not move. Such points are called nodes. Midway between each pair of nodes is a point of maximum amplitude of vibration called an antinode.

String fixed at both ends (Figure 16-11 of Tipler):

The standing wave condition is

$$L = n\frac{\lambda_n}{2}, \quad n = 1, 2, 3, \ldots$$

and with $f_n \lambda_n = v$ the resonance frequencies become

$$f_n = n\frac{v}{2L} = n f_1, \quad n = 1, 2, 3, \ldots$$

where f_1 is the fundamental frequency. Example: pianos.

String fixed at one end (Figure 16-14 of Tipler):

The free end is an antinode. The standing wave condition can thus be written

$$L = n \frac{\lambda_n}{4}, \quad n = 1, 3, 5, \ldots$$

and with $f_n \lambda_n = v$ the resonance frequencies become

$$f_n = n \frac{v}{4L} = n f_1, \quad n = 1, 3, 5, \ldots$$

where f_1 is the fundamental frequency.

Example: The air column in an organ pipe.

Exercises:

A pipe, closed at one end, has successive resonance frequencies 700 Hz, 900 Hz, 1100 Hz, ... Find the fundamental frequency! (It is 100 Hz).

Assume, the speed of sound is 340 m/s. What is the length of the pipe?

Thermodynamics (Chapters 18–21)

Temperature and Kinetic Gas Theory

The temperature of an object is a measure of its average internal molecular kinetic energy. A consistent temperature scale can be defined in terms of properties of gases at low densities.

Thermal Equilibrium and Temperature:

If we place a warm and a cold object into thermal contact, the cold object warms and the warm object cools. Eventually, this process stops and the two objects are said to be in thermal equilibrium and the two objects are then defined to have the same temperature.

The Celsius and the Kelvin Temperature Scales:

The ice point temperature or normal freezing point of water is the freezing point of water at a pressure of $1\,\mathrm{atm}$. The steam-point temperature or normal boiling point of water is the the boiling point of water at a pressure of $1\,\mathrm{atm}$.

Units of pressure: Pascal (Pa): $1\,\mathrm{Pa} = 1\,\mathrm{N/m^2}$, atmosphere (atm): $1\,\mathrm{atm} = 101.325\,\mathrm{kPa}$.

Gas Thermometers:

For the constant-volume gas thermometers the measured temperatures agree closely even far from the calibration points. The change in gas pressure is used to measure the temperature (Figure 18-3 of Tipler). For the Celsius scale:

$$t_{\mathrm{C}} = \frac{P_t - P_0}{P_{100} - P_0} \times 100^{\circ}\mathrm{C}\ .$$

The agreement between different gas thermometers is best at low gas densities.

Figure 18-6 of Tipler shows a plot of measured temperature versus pressure for a constant-volume gas thermometer at very low density. When we extrapolate the straight line towards zero pressure, the temperature

approaches the absolute zero at

$$T_0 = -273.15°\text{C} \ .$$

A reference point that is more precisely reproducible than either the ice or the steam points is the triple point of water. This is the unique temperature and pressure at which water, water vapor and ice coexist in equilibrium, namely at $0.01°\text{C}$ and $P_3 = 4.58\,\text{mmHg}$. The Kelvin or ideal gas temperature scale is defined so that the triple point is at $273.16\,\text{K}$ and the absolute zero temperature is at $0\,\text{K}$:

$$T = \frac{273.16\ K}{P_3} P$$

where P is the observed pressure of the gas thermometer. To convert from Celsius to Kelvin one merely adjust the absolute zero temperature:

$$T = t_\text{C}\,\text{K/C} + 273.15\,\text{K} \ .$$

The Ideal-Gas Law

Experimental results by Robert Boyle (1627-1691), Jacques Charles (1746-1823) and Gay-Lussac (1778-1850) for a fixed amount of gas combine to the equation

$$P V = C T$$

where P is the pressure, V is the volume, T is the absolute temperature and C is a proportionality constant. We see that C is proportional to the volume by doubling the amount of gas, while holding the pressure and temperature constant. Therefore, we write $C = k N$, where k is a constant and N the number of molecules in the gas. Then, we have

$$P V = N k T \ .$$

The constant k is called Boltzmann constant and found experimentally to be

$$k = 1.381 \times 10^{-23} \, \text{J/K} = 8.617 \times 10^{-5} \, \text{ev/K}$$

An amount of gas is often expressed in moles. A mole (mol) is the amount of a substance that contains Avogadro's number N_A of molecule, defined as the number of carbon atoms in $12\,\mathrm{g}$ of ^{12}C:

$$N_A = 6.022 \times 10^{23} \ .$$

If we have n moles of a substance, the number of molecules is then $N = n\,N_A$. The constant

$$R = N_A\,k = 8.3145\,\mathrm{J/mol \cdot K} = 0.08206\,\mathrm{L \cdot atm/mol \cdot K}$$

is called universal gas constant. The pressure, volume, and temperature of an ideal gas are related by

$$P\,V = n\,N_A\,k\,T = n\,R\,T \ .$$

The variables P, V, and T describe the macroscopic state of a gas. The microscopic state would be described by giving the position and velocity of each molecule of the gas.

Heat (Chapter 19)

In the 1840s James Joule showed that heat is a form of energy, and energy is conserved.

Heat Capacity and Specific Heat

When heat energy flows into a substance, the temperature of the substance rises unless the substance undergoes a phase transition. The amount of heat energy Q needed to raise the temperature of a substance by a small amount is proportional to the temperature change and to the mass of the substance:

$$Q = C \, \triangle T = m \, c \, \triangle T$$

where C is called heat capacity and c specific heat. The heat capacity is energy needed per one degree temperature change, and the specific heat is the heat capacity per unit mass:

$$c = \frac{C}{m} \, .$$

The calorie (cal) is the historical unit of heat energy. Nowadays the calorie is defined in terms of the SI energy unit:

$$1\,\text{cal} = 4.184\,\text{J}\ .$$

Change of Phase and Latent Heat

When heat is added to ice at $0°C$, the temperature of the ice does not change, instead it melts. This is an example of a phase change. Examples of phase changes are:

1. Fusion (liquid to solid)

2. Melting (solid to liquid)

3. Vaporization (liquid to vapor or gas)

4. Condensation (vapor or gas to liquid)

The fact that the temperature remains constant during a phase change has to do with the potential energy of the molecules. For instance, the molecules in a liquid are close together and exert attractive forces on each other, while the molecules in a gas are far apart. Vaporization achieves the corresponding increase in potential energy.

The heat energy pre unit mass required for a change of phase is called the latent heat L:

$$Q = m\,L \ .$$

The latent heat for melting is called latent heat of fusion. Example: For water at a pressure of $1\,\text{atm}$ the latent heat of fusion is $L_f = 79.7\,\text{kcal/kg} = 333.5\,\text{kJ/kg}$ and the latent heat of vaporization is $L_v = 540\,\text{kcal/kg} = 2.26\,\text{MJ/kg}$.

Exercises:

Convert $20°C$ into Kelvin.

The specific heat of a liquid is $c_{li} = 0.5\,\mathrm{J/(g \cdot K)}$. The liquid is initially at some temperature $T_i = 300\,\mathrm{K}$ and heated to the boiling point at $T_{bp} = 330\,\mathrm{K}$. Calculate the amount of heat required for $500\,\mathrm{g}$ of the liquid.

How much energy is then needed to vaporize the liquid, if its latent heat of vaporization is $100\,\mathrm{J/g}$?

Suppose the vapour phase can be treated as an ideal gas and we have $11\,\mathrm{mols}$ of the gas at a pressure of $100\,\mathrm{kPa}$. What is the volume of the gas at the boiling point?