ADVANCED DYNAMICS — PHY–4241

Final Exam

April 28, 2003

The final exam consists of two problems. You must select one and only one of them; no extra (nor partial) credit will be given for solving both. You have exactly two hours to complete the exam. You may **NOT** consult any books or notes; you will only need pencil and paper to solve the exam. Please write clearly! **Best of luck and have a great summer**.

Academic Honor Code: *Students are expected to uphold the Academic Honor Code published in the Florida State University Bulletin and the Student Handbook. The first paragraph reads: The Academic Honor System of Florida State University is based on the premise that each student has the responsibility (1) to uphold the highest standards of academic integrity in the student's own work, (2) to refuse to tolerate violations of academic integrity in the University community, and (3) to foster a high sense of integrity and social responsibility on the part of the University community.*

Student's Name:
Student's Social Security Number:

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Student's Signature
I have upheld the Academic Honor Code.

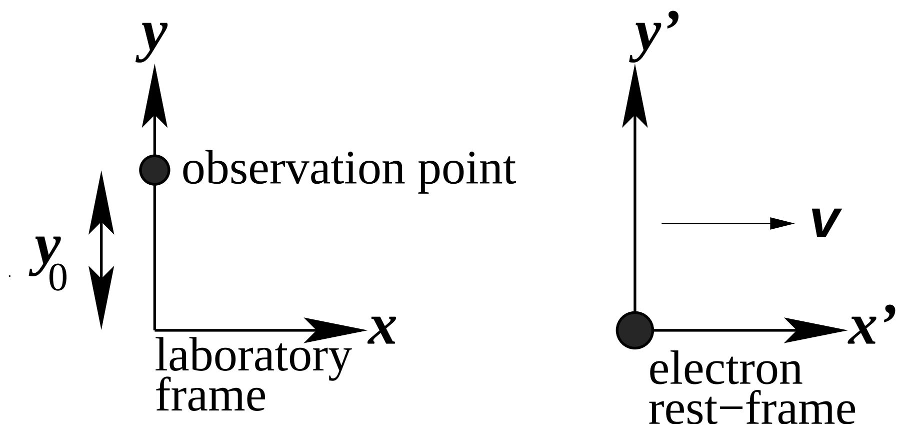
PROBLEM 1

An electron (of charge e) is traveling at a constant velocity $\mathbf{v} = v\hat{\mathbf{x}}$ along the x-axis with respect to the laboratory frame. You may assume that at time $t=0$ the electron passes through the origin of the laboratory frame.

(a) Compute the electric $\mathbf{E}'(\mathbf{r}', t')$ and magnetic $\mathbf{B}'(\mathbf{r}', t')$ fields in the rest frame of the electron, *i.e.,* as measured by an observer in an inertial frame traveling with the electron.

(b) From the above configuration of fields, construct the electromagnetic field tensor $F'_{\mu\nu}(\mathbf{r}', t')$ as measured by an observer in an inertial frame traveling with the electron.

(c) Use the appropriate Lorentz transformation on the electromagnetic field tensor to extract the electric $\mathbf{E}(\mathbf{r}_0, t)$ and magnetic $\mathbf{B}(\mathbf{r}_0, t)$ fields measured by an observer in the laboratory frame located at the point $\mathbf{r}_0 = (0, y_0, 0)$ (see the figure).



PROBLEM 2

Three identical mass points of mass M, constrained to move in a circle, are connected to each other and to a fixed support by means of four identical springs of spring constant k, as indicated in the figure.

(a) Obtain the Lagrangian of the system in the limit of small oscillations. That is, compute the 3×3 symmetric matrices $\hat{\mathcal{T}}$ and $\hat{\mathcal{V}}$ such that the Lagrangian may be written as

$$L = \frac{1}{2}\dot{\eta}^T \hat{\mathcal{T}} \dot{\eta} - \frac{1}{2}\eta^T \hat{\mathcal{V}} \eta \,, \quad \Big(\eta^T \equiv (x_1, x_2, x_3)\Big) \,,$$

where x_n is the displacement of the nth mass point from to its equilibrium position.

(b) Obtain the normal frequencies (eigenvalues) and normal modes (eigenvectors) of the problem. Note that there is no need to normalize the eigenvectors.

(c) Assuming that particle number 1 is displaced by one unit from its equilibrium position at time $t = 0$ (that is, $x_1(0) = 1$ and $x_2(0) = x_3(0) = \dot{x}_1(0) = \dot{x}_2(0) = \dot{x}_3(0) = 0$) compute the coordinates of all three particles as a function of time.