ADVANCED DYNAMICS — PHY–4241/5227

First Midterm Exam

February 17, 2003

PROBLEM 1

Write the principle of least action and the Euler-Lagrange equation(s) of motion for a 1-dimensional Lagrangian of the form:

$$L = \frac{1}{2}m\dot{x}^2 - V(x) \ .$$

Is the resulting equation consistent with Newton's second law?

PROBLEM 2

Consider the 3-dimensional Lagrangian for a particle of mass m moving in the presence of a spherically symmetric potential $V(\mathbf{r})=V(r)$. That is,

$$L = \frac{1}{2}m\Big(\dot{r}^2 + r^2\dot{\theta}^2 + r^2\sin^2\theta\dot{\phi}^2\Big) - V(r) \ .$$

Identify as many conserved quantities (*i.e.,* constants of the motion) as you can from the mere structure of the Lagrangian.

PROBLEM 3

The Hamiltonian for a one-dimensional harmonic oscillator of mass $m=1$ and spring constant $k=1$ is given by the following expression:

$$H = \frac{p^2}{2} + \frac{x^2}{2} \ .$$

Compute the Poisson bracket of the function $f(x,p;t)=x^2$ with the Hamiltonian. Is $f(x,p;t)$ a constant of the motion? Explain.

PROBLEM 4

Write the four Maxwell's equations (in vacuum) in the presence of a charge density ρ and a current density $\mathbf{J}$ (you may use SI or Gaussian units).

PROBLEM 5

Before Maxwell's contribution to electrodynamics, Ampere's law (in Gaussian units) was given by:

$$\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{J} \ .$$

Introduce Maxwell's correction to Ampere's law and discuss briefly why it proved to be such a fundamental contribution.

PROBLEM 6

Discuss briefly the possibility or impossibility of the following configurations of electric and magnetic fields.

$$\text{a)}\ \mathbf{E} = \frac{\alpha}{r^2}\hat{\mathbf{r}} \quad \text{and} \quad \mathbf{B} = 0 \ .$$

$$\text{b)}\ \mathbf{B} = \frac{\alpha}{r^2}\hat{\mathbf{r}} \quad \text{and} \quad \mathbf{E} = 0 \ .$$