ADVANCED DYNAMICS — PHY–4241/5227

Second Midterm Exam

April 7, 2003

PROBLEM 1

Write the two basic postulates of the Special Theory of Relativity.

PROBLEM 2

A bullet is fired from a train at a velocity v_b (to the right) and rapidity ξ_b. The train itself is moving relative to a stationary platform at a velocity v_t (also to the right) and rapidity ξ_t. What are the velocity and rapidity of the bullet as measured by an observer standing in the platform.

PROBLEM 3

Consider an event A that happens at a space-time point $(x_A = 5, y_A = 3, z_A = 0)$ and at a time t_A given by $ct_A = 15$, and an event B that occurs at $(10, 8, 5)$ and $ct_B = 5$, both in system $\mathcal{S}$.

(i) What is the invariant interval between A and B?

(ii) Is there an inertial frame in which they occur simultaneously? Answer yes or no and explain your answer but do **NOT** find the frame even if one exists.

(iii) Is there an inertial frame in which they occur at the same point? Answer yes or no and explain your answer but do **NOT** find the frame even if one exists.

PROBLEM 4

Consider a massless ($m = 0$) photon traveling through empty space with a zero linear momentum ($\mathbf{p} = \mathbf{0}$).

(i) What is the magnitude of the velocity of the photon?

(ii) What is the energy of the photon?

PROBLEM 5

Consider an electron of charge e and mass m traveling unperturbed through space with linear momentum $\mathbf{p}$ and energy $E = \sqrt{(\mathbf{p}c)^2 + (mc^2)^2}$. How do these four quantities (charge, mass, linear momentum, and energy) transform under a Lorentz transformation?

PROBLEM 6

Consider the head-on collision of an electron and a positron (identical in all respects except for their opposite charges) each with energy $E = 1$ GeV. In the center-of-momentum frame their respective four-momenta are given by:

$$p^\mu_{e^+} = (E, +pc) \; ; \quad p^\mu_{e^-} = (E, -pc) \; ,$$

where $pc = \sqrt{E^2 - (mc^2)^2}$. The Mandelstam variable s for this collision is given by

$$s = (p_{e^+} + p_{e^-}) \cdot (p_{e^+} + p_{e^-}) \equiv (p_{e^+} + p_{e^-})^\mu (p_{e^+} + p_{e^-})_\mu = 4E^2 = 4 \text{ GeV}^2 \; .$$

(i) What is the numerical value of s in the rest-frame of the positron?

(ii) What is the numerical value of s in the rest-frame of the electron?