MIDTERM ADVANCED DYNAMICS — PHY–4241/5227

March 1, 2004

PROBLEM 1 (20 points)

The Lagrangian of the 1D harmonic oscillator is

$$L = \frac{1}{2}\, m\, \dot{x}^2 - \frac{1}{2}\, k\, x^2 \ .$$

1. Find the momentum p.
2. Write down the Hamiltonian $H(p, x)$.
3. Write down Hamilton's equations.
4. Show that Hamilton's equations give Newton's force law.

PROBLEM 2 (20 points)

1. Use spherical coordinates (θ, ϕ, r) to write down the Lagrangian for a particle of mass m moving in the presence of a spherically symmetric potential $V(\mathbf{r}) = V(r)$.
2. Write down the Euler-Lagrange equations for the two angles.
3. Use the Lagrangian to identify two conserved quantities.
4. Find $\phi(t)$ for the solution with the initial condition $\dot{\phi}(0) = 0$ where ϕ is the azimuth angle.

PROBLEM 3 (5 points)

Write down the four Maxwell's equations (in vacuum) in the presence of a charge density ρ and a current density $\mathbf{J}$ (you may use SI or Gaussian units).

PROBLEM 4 (15 points)

1. Write down the two basic postulates of the Special Theory of Relativity.
2. The lifetime of a pion is $\tau = 2.6 \times 10^{-8}$ s (with respect to its rest frame). Assume the pion travels in a collider experiment at 99% of the speed of light with respect to the Lab frame.
   (a) How long is the observed lifetime of the pion in the Lab frame?
   (b) How far does the pion travel in the Lab frame?