ADVANCED MECHANICS — PHY–4241/5227

HOMEWORK 4

(January 26, 2004)
Due Monday, February 2, 2004 (late afternoon)

PROBLEM 9

A ladder of length l and mass M leans with one end against a vertical wall and stands with its other end on a surface, which is given by an equation $y = f(x)$. Neglect friction. For which function $f(x)$ is the ladder at all positions in static equilibrium?

Hints: Approxiate the ladder by two masspoints of mass $m = M/2$ at its endpoints and use Lagrange multipliers to incorparate the constraints in the equations of motion.

PROBLEM 10

Consider the spherical pendulum of mass m again and (figure 7-10 of M& T).

a) Use the Legendre transformation

$$H(\theta, \phi, p_\theta, p_\phi) = \dot{\theta} p_\theta + \dot{\phi} p_\phi - L(\theta, \phi, \dot{\theta}, \dot{\phi}) ,$$

to construct the Hamiltonian of the system and show that it is identical to the energy $E{=}T{+}V$.

b) Write down Hamilton's equations of motions for the system and identify a conserved quantity.

PROBLEM 11

a) Show that if f, g, and h are arbitrary functions of q and p (and possible time) then the following relations among Poisson brackets hold true:

$$\begin{aligned} [fg, h] &= [f, h]g + f[g, h] , \\ [f, gh] &= [f, g]h + g[f, h] . \end{aligned}$$

b) Show that the Poisson bracket between an **arbitrary** function of $r = \sqrt{(x^2 + y^2 + z^2)}$ and the momentum $\mathbf{p}$ is given by

$$[f(r), p_i] = \frac{x_i}{r} f'(r) .$$

c) Assuming the validity of the fundamental Poisson-bracket relations between coordinates and momenta

$$[x_i, x_j] = [p_i, p_j] = 0 ; \quad [x_i, p_j] = \delta_{ij} ,$$

compute the following Poisson brackets:

$$[x_i, L_j] , \ [p_i, L_j] , \text{ and } [L_i, L_j] ,$$

where, using the summation convention, $L_j = \epsilon_{jkl}\, x_k\, p_l$ is the ith component of the angular momentum of the system.