ADVANCED DYNAMICS — PHY–4241/5227

HOMEWORK 6

(February 11, 2004)

Due Monday, February 16, 2004 (late afternoon)

PROBLEM 15

Continue with problem 13 and find a Galilei transformation $\vec{r}^{\,\prime} = \vec{r} + \vec{v}_0\, t$ under which the electric field becomes zero. Hint: In the new reference frame the Lorentz force has the form

$$\vec{F}^{\,\prime} = q\,\left(\vec{E}^{\,\prime} + c^{-1}\,\dot{\vec{r}}^{\,\prime} \times \vec{B}^{\,\prime}\right)\ .$$

PROBLEM 16

By using the following definition for the antisymmetric **Levi-Civita** symbol:

$$\varepsilon_{ijk} = \begin{cases} +1, & \text{if } ijk \text{ is an even permutation of 123;} \\ -1, & \text{if } ijk \text{ is an odd permutation of 123;} \\ 0, & \text{otherwise;} \end{cases}$$

show, in a line or two, that

a) $\nabla \times \Big(\nabla\Phi(\mathbf{x})\Big) = 0$.

b) $\nabla \cdot \Big(\nabla \times \mathbf{A}(\mathbf{x})\Big) = 0$.

c) $\nabla \times \Big(\nabla \times \mathbf{A}(\mathbf{x})\Big) = \nabla\Big(\nabla \cdot \mathbf{A}(\mathbf{x})\Big) - \nabla^2\mathbf{A}(\mathbf{x})$.

You may find useful the following two important identities:

$$\begin{aligned} \varepsilon_{ijk}\varepsilon_{ijl} &= 2\delta_{kl}\ , \\ \varepsilon_{ijk}\varepsilon_{ilm} &= \delta_{jl}\delta_{km} - \delta_{jm}\delta_{kl}\ . \end{aligned}$$