ADVANCED DYNAMICS — PHY–4241/5227

HOMEWORK 9

(March 1, 2004)

Due Monday, March 14, 2004 (after springbreak, late afternoon)

PROBLEM 20 (continuation of 18)

How far away from earth did the spaceship travel? Express the result in light years.

1. After which time (on its own clock) did the spaceship reach 90% of the speed of light with respect to earth?
2. When does a message sent from earth in 2025 (precisely five years after take off) reach the spaceship occupants? Answer this question
   (a) with respect to the time on earth and
   (b) with respect to the clock on the spaceship.

PROBLEM 21 (continuation of 20)

Assume that the spaceship moves by exhausting particles at the speed $v = 0.66c$.

1. Derive an expression for $m(\tau)$, the (remaining) mass of the spaceship at proper time τ.
2. Which fraction of the original mass is left, after the spacetrip has been performed?
3. Repeat the consideration of (1) and (2) under the assumption that, instead of exhausting particles, the spaceship moves now by radiating mass into light.

PROBLEM 22

An electron and a positron, each with mass equal to 0.511 MeV, annihilate at rest into two photons. Chose the rest frame of the positronium, such that one of the photons moves in positive $x = x^1$ direction.

1. For each photon find the momentum 4-vector.
2. Find the 4-momentum in a frame that moves at a velocity $c\beta\hat{x}$ with respect to the rest frame of the positronium.
3. Suppose that the annihilation took place 10^9 years ago in a galaxy that is receding from us at $\beta = \frac{4}{5}$. What is the energy of the photon that we observe?

Turn Over

PROBLEM 23

The matrix L is defined by

$$L = \begin{pmatrix} l^0{}_0 & l^0{}_1 & l^0{}_2 & l^0{}_3 \\ l^1{}_0 & l^1{}_1 & l^1{}_2 & l^1{}_3 \\ l^2{}_0 & l^2{}_1 & l^2{}_2 & l^2{}_3 \\ l^3{}_0 & l^3{}_1 & l^3{}_2 & l^3{}_3 \end{pmatrix}$$

1. Calculate $-g\,L$.
2. Write down the transpose matrix $\tilde{L}\dagger$.
3. Calculate $\tilde{L}\,g$.
4. Compare (a) and (c) to find the general form of L (*i.e.* use $\tilde{L}\,g = -g\,L$).
5. Obtain the same result by discussing the elements of the equation $g^{\alpha\beta}\tilde{l}_\beta{}^\gamma g_{\gamma\delta} = -l^\alpha{}_\delta$. (Hint: do the contractions first.)