ADVANCED MECHANICS — PHY–4241/5227

HOMEWORK 1

(January 5, 2003)
Due on Monday, January 13, 2003

PROBLEM 1

Read, and read again and again, **"The Principle of Least Action"** from Chapter 19 of *"The Feynman Lectures on Physics", Vol. II.* A true jewel written by the best physicist of the second half of the twentieth century; Enjoy!

PROBLEM 2

Consider a particle of mass $m \equiv 1$ moving, from $x_1 = 0$ at time $t_1 = 0$ to $x_2 = 1$ at time $t_2 = \pi/2$, under the influence of a one-dimensional harmonic potential of the form:

$$V(x) = \frac{1}{2}x^2 \ .$$

a) Using Euler-Lagrange's equations of motion, obtain the time-dependent motion of the system; *i.e.,* solve for $x(t)$. Compute the action for this exact path.

b) Using an approximate linear path of the form $x(t) = a + bt$, compute the action for this path and compare it with the exact value obtained in part a).

c) Using an approximate quadratic path of the form $x(t) = a + bt + ct^2$, compute the action for this path and compare it with the exact value obtained in part a).

Hint: For parts b) and c) make sure that the paths are consistent with the boundary conditions. If any constant remains undetermined, fix it by minimizing the action.

PROBLEM 3

A projectile of mass M is fired at $t = 0$ from a cannon located at the origin of the coordinate system $(x(0) = y(0) = 0)$ with a velocity v_0 directed at an angle of $\theta_0 \neq 0$ relative to the horizontal direction. The projectile lands after a time T at a horizontal distance L away from the origin, *i.e.,* $x(T) = L$ and $y(T) = 0$.

a) Write the Lagrangian for the system and, using Euler-Lagrange's equations of motion, solve for the trajectory of the projectile. In particular, obtain the values of the initial velocity v_0 and θ_0 necessary for the projectile to follow the prescribed trajectory.

b) Compute the action for the exact trajectory obtained in part a).

c) Using the exact value of $x(t)$ obtained in part a) but now approximating $y(t)$ by a sinusoidal function of the form

$$y(t) = A\sin(\pi\frac{t}{T}) \ ,$$

compute the action for this approximate path. Determine the amplitude A by demanding that the action be minimized with respect to A.

d) Make a plot of $y(t)$ for the exact and for the (optimal) sinusoidal paths (using the values of $g = 9.81\mathrm{m/s}^2$ and $T = 10$ s. Compute the exact and approximate actions for these values of the parameters.