ADVANCED MECHANICS — PHY–4241/5227

HOMEWORK 2

(January 12, 2003)
Due on Monday, January 20, 2003

PROBLEM 4

Consider a particle of mass $m \equiv 1$ moving, from $x_1 = 0$ at time $t_1 = 0$ to $x_2 = 1$ at time $t_2 = \pi/2$, under the influence of a one-dimensional anharmonic potential of the form:

$$V(x) = \frac{1}{2}x^2 + \frac{\lambda}{4}x^4 \,, \quad (\lambda \equiv \frac{1}{4}) \,.$$

a) Using Euler-Lagrange's equations of motion, obtain—but do not solve—the non-linear differential equation for $x(t)$.

b) Propose an approximate solution of the form $x(t) = \sin(t) + \alpha \sin(2t)$ and fix the value of the "variational" parameter α by minimizing the action. Compare the action for the optimal path and for the path having $\alpha = 0$.

c) Using the $\alpha = 0$ path and the optimal path obtained in part b), plot the energy of the system. Is the energy conserved? Explain.

PROBLEM 5

A particle of mass m is constrained to move on a massless hoop of radius a fixed in a vertical plane that is rotating about the vertical axis with constant angular speed ω. The particle can slide through the hoop under the action of gravity.

a) Obtain the Lagrangian of the system using the polar angle θ as the sole generalized coordinate.

b) Compute the Hamiltonian function of the system and argue that it is a constant of the motion. In particular show that it may be written as follows:

$$h = \frac{1}{2}ma^2\dot{\theta}^2 + V_{\text{eff}}(\theta) \,.$$

Compute explicitly the value of $V_{\text{eff}}(\theta)$.

c) Make a plot of $V_{\text{eff}}(\theta)$ and use it to show that if ω is greater than a minimum value ω_0, there can be a solution in which the particle remains stationary at a point other than the bottom of the hoop. Find this stationary point and the value of ω_0.

PROBLEM 6

(Problem 7.7 Marion and Thornton)

A double pendulum consists of two simple pendula, with one pendulum suspended from the bob of the other. The two pendula have equal length l and have bobs of equal mass m; both pendula are confined to move in the same plane.

a) Identify clearly the number of independent generalized coordinates. In an effort to maintain uniformity of notation, label the generalized coordinates as $\theta_1, \theta_2, \ldots, \theta_N$.

b) Write the Lagrangian of the system in terms of the generalized coordinates identified in part a).

c) Compute—without solving—the Euler-Lagrange's equations of motion for the system. Do not assume small angles.