ADVANCED DYNAMICS — PHY–4241/5227

HOMEWORK 4

(January 25, 2003)
Due on Monday, February 3, 2003

PROBLEM 10

A particle of mass m and electric charge q moves under the influence of a constant magnetic field of the form $\mathbf{B}(\mathbf{r}) = B_0\hat{\mathbf{z}}$. Obtain the most general solution for the velocity $\mathbf{v}(t)$ using Newton's second law of motion in combination with the Lorentz force. That is,

$$\mathbf{F} = m\dot{\mathbf{v}} = \frac{q}{c}\mathbf{v} \times \mathbf{B} \ .$$

PROBLEM 11 and 12

The Lagrangian for a particle of mass m and electric charge q moving under the influence of a magnetic (but no electric) field is given by

$$L(\mathbf{r}, \dot{\mathbf{r}}) = \frac{1}{2}m\dot{\mathbf{r}}^2 + \frac{q}{c}\dot{\mathbf{r}} \cdot \mathbf{A}(\mathbf{r}) \ ,$$

where $\mathbf{A}(\mathbf{r})$ is the vector potential. Assume that the magnetic field is constant and given by $\mathbf{B}(\mathbf{r}) = B_0\hat{\mathbf{z}}$.

a) Show that for such a constant magnetic field the vector potential can be written in the form $\mathbf{A}(\mathbf{r}) = \frac{1}{2}(\mathbf{B} \times \mathbf{r})$. That is, show that such a vector potential satisfies: $\nabla \times \mathbf{A} = \mathbf{B}$.

b) Construct the Hamiltonian of the system in terms of the Cartesian coordinates and the corresponding canonical momenta of the particle.

c) Denoting by $\boldsymbol{\pi} \equiv m\dot{\mathbf{r}} = m\mathbf{v}$ the "mechanical" momentum of the particle, evaluate the following Poisson brackets:

$$[\pi_x, \pi_y] \ , \ [\pi_y, \pi_z] \ , \ [\pi_z, \pi_x] \ .$$

d) By re-expressing the Hamiltonian of part b) in terms of the mechanical momentum of the particle—and by using the results derived in part c)—obtain the most general solution for $\boldsymbol{\pi}(t)$ by using Poisson's equation:

$$\frac{d\boldsymbol{\pi}}{dt} = [\boldsymbol{\pi}, H] \ .$$

Interpret your results on the basis of a "conventional" (Newton's second law plus Lorentz force) approach (see Problem 10).