ADVANCED DYNAMICS — PHY–4241/5227

HOMEWORK 7

(February 16, 2003)
Due on Monday, March 3, 2003

PROBLEM 19

Read and summarize briefly, in approximately five lines, the enclosed article entitled "*Experimental Verification of a Negative Index of Refraction*" by Shelby, Smith, and Schultz. This article was selected as part of **Physics News in 2001**, a publication of the American Physical Society devoted to the most important developments in Physics during 2001 (document also enclosed).

PROBLEM 20

Consider an electromagnetic plane wave incident on an interface that separates two media, as we did in class. Now, however, let the electric (not the magnetic) field be perpendicular to the plane of incidence.

a) Write expressions for the wavenumber $\mathbf{k}$, the electric field $\mathbf{E}$, and the magnetic field $\mathbf{B}$ for the incident, reflected, and transmitted wave.

b) Write the four boundary conditions satisfied by the fields. Show that only two of them are independent.

c) By solving the above equations compute the reflections and transmission intensities. Make sure that they satisfy $R+T=1$.

d) Make a plot of the reflection and transmission intensities as a function of the incident angle using $n_1=1$, $n_2=1.5$, and $\mu_1=\mu_2=1$. Is there a Brewster's angle for this problem? That is, is there an incident angle for which R vanishes for this configuration of fields?

PROBLEM 21

Consider a linear medium of dielectric constant ϵ and permeability μ. Further, assume that the energy density (*i.e.,* the energy per unit volume) stored in space is given by:

$$u = \frac{1}{8\pi}\left(\mathbf{E}\cdot\mathbf{D} + \mathbf{B}\cdot\mathbf{H}\right) \; .$$

a) Starting from the above equation, and using Maxwell's equations, show that the energy satisfies a continuity like equation of the form:

$$\frac{\partial u}{\partial t} + \nabla\cdot\mathbf{S} = -\mathbf{J}\cdot\mathbf{E} \; .$$

Display an explicit form for the *Poynting* vector $\mathbf{S}$ in terms of the electromagnetic fields. What is the physical significance of the $\mathbf{J}\cdot\mathbf{E}$ term?

b) Compute the *time-averaged* Poynting vector for an electromagnetic plane wave of wave number $\mathbf{k}$ and frequency ω. Note that the definition of the time-averaged Poynting vector is given by

$$\langle \mathbf{S} \rangle_{\text{avg}} = \frac{1}{T} \int_0^T \mathbf{S}(\mathbf{x}, t) dt \ , \qquad (T = 2\pi/\omega) \ .$$

PROBLEM 22

As a simple preamble to the **Special Theory of Relativity**, consider an electron moving along the axis of a parallel-plate capacitor that provides a constant electric field of magnitude $E_0 = 10^6 \text{V/m}$. Compute how long will it take for this electron to reach the speed of light if it obeys Newton's 2nd law of motion.