ADVANCED DYNAMICS — PHY 4241/5227

HOME AND CLASS WORK – SET 2

(January 12, 2009)

(8a) Derive the Euler-Lagrange equations from

$$\delta \int_{t_1}^{t_2} dt\, L = 0\,, \qquad L = L(q_i, \dot{q}_i, t)\,, \qquad i = 1, \dots, n_f\,.$$

Due January 14 in class (6 points).

(8b) Write down and solve the Euler-Lagrange equation for a free particle, $T = m\dot{x}^2/2$, $U = 0$. Due January 14 in class (2 points).

(9) Write down and solve the Euler-Lagrange equations for

1. The one-dimensional harmonic oscillator $T = m\dot{x}^2/2$, $U = k\,x^2/2$.
2. An unphysical system with $T = 0$, $U = x^2/2$. Is the action a minimum?
3. An unphysical system with $T = 0$, $U = -x^2/2$. Is the action a mimimum?
4. An unphysical system with $T = 0$, $U = x$.
5. The three-dimensional harmonic oscillator $T = m\vec{v}^{\,2}/2$, $U = k\,\vec{r}^{\,2}/2$.
6. Circular motion without friction $T = m\,l^2\,\dot{\theta}^{\,2}/2$, $U = \text{const}$.

Due January 16 in class (6 points).

(10) Consider the spherical pendulum (a point mass m on the surface of a sphere of radius R under the influence of gravity $-g\hat{z}$).

1. Write down the Lagrange function using spherical coordinates.
2. Find the Euler-Lagrange equations.
3. Calculate the special solutions for $\theta = \text{constant}$. Describe this motion.

Due January 26 in class (6 points).

(11) A double pendulum consists of two simple pendula, with one pendulum suspended from the bob of the other. Assume that the two pendula have equal lengths, have bobs of equal mass and are confined to move in the same plane.

1. Define angles ϕ and ψ for the pendula with respect to the gravity direction and write down the Lagrange function.
2. Derive the equations of motion for small oscillations around the rest position $\phi = \psi = 0$.
3. Calculate the normal modes.

Due January 26 before class (10 points).