ADVANCED DYNAMICS — PHY 4241/5227

HOME AND CLASS WORK – SET 4

(January 27, 2009)

(15) A homogeneous cylinder of mass M roles down and inclined plane. Use a Lagrange multiplier to find the constraint force, which prevent the cylinder from sliding (suitable coordinates will be given in class). Due January 26 in class (5 points).

(16) Derive (due January 28 in class, 8 points)

$$\frac{d}{dt}\left(L - \sum_j \dot{q}_j \frac{\partial L}{\partial \dot{q}_j}\right) = 0 \text{ from } \delta_t L = \frac{\partial L}{\partial t}\,\delta t = 0 \ .$$

Assume a bilinear kinetic Energy

$$T = \sum_{j,k} a_{ij}\,\dot{q}_j \dot{q}_k \text{ and prove } \sum_i \frac{T}{\dot{q}_i} = 2T \ .$$

Read M&T chapter 7.9 (due January 28).

(17) Legendre transformation: Define the Hamiltonian by

$$H = \left(\sum_j \dot{q}_j \frac{\partial L}{\partial \dot{q}_j} - L\right) \text{ and the generalized momentum by } p_j = \frac{\partial L}{\partial \dot{q}_j} \ .$$

Show that the Hamiltonian is a function of q_j and p_j only: $H = H(q_j, p_j)$. Then derive Hamilton's equations of motion. Hint: Calculate dH. Due January 30 in class (6 points).

(18) Make a table of the nine scalar products of the cartesian unit vectors $\widehat{x}$, $\widehat{y}$, $\widehat{z}$ with the spherical unit vectors $\widehat{r}$, $\widehat{\theta}$, $\widehat{\phi}$. Due February 2 before class (6 points).

(19) Calculate the velocity

$$\vec{v} = \frac{d}{dt}\vec{r} = \frac{d}{dt}(r\widehat{r})$$

algebraically along the following lines:

(a) Calculate

$$\frac{d}{dt}\widehat{r}$$

as linear combination in the basis $\widehat{x}$, $\widehat{y}$ and $\widehat{z}$ (5 points).

(b) Expand $\widehat{x}$, $\widehat{y}$ and $\widehat{z}$ in terms of $\widehat{r}$, $\widehat{\theta}$ and $\widehat{\phi}$. Use trigonometric relations to simplify the result (5 points).

Due February 4 before class.