ADVANCED DYNAMICS — PHY 4241/5227

HOME AND CLASS WORK – SET 6

(February 11, 2009)

(24) Poisson brackets are defined by

$$[g, h] = \sum_k \left(\frac{\partial g}{\partial q_k} \frac{\partial h}{\partial p_k} - \frac{\partial h}{\partial q_k} \frac{\partial g}{\partial p_k} \right)$$

where g and h are functions of q_i, p_i and, possibly, t. Show the following properties:

1. $$\frac{dg}{dt} = [g, H] + \frac{\partial g}{\partial t} \ .$$

2. $$\dot{q}_j = [q_j, H] \ .$$

3. $$\dot{p}_j = [p_j, H] \ .$$

4. $$[x_i, x_j] = [p_i, p_j] = 0 \ ; \quad [x_i, p_j] = \delta_{ij} \ ,$$

5. $$[x_i, L_j] = \epsilon_{ijk} x_k \ , \ [p_i, L_j] = \epsilon_{ijk} p_k \ , \text{ and } \ [L_i, L_j] = \epsilon_{ijk} L_k \ , \ \ ,$$
   where the Einstein summation convention is used and $L_j = \epsilon_{jkl}\, x_k\, p_l$ is the ith component of the angular momentum of the system.

Due February 13 before class (10 points).

(25) Consider two point particles with a Lagrangian (due February 13 in class)

$$\mathcal{L} = \frac{m_1}{2} \vec{v}_1^{\,2} + \frac{m_2}{2} \vec{v}_2^{\,2} - U(r) \,, \quad r = |\vec{r}| \,, \ \vec{r} = \vec{r}_1 - \vec{r} \ .$$

1. Is the energy of this system conserved (with reason) (2 points)?
2. Define the center of mass vector $\vec{R} = \vec{R}(t)$ by $M\dot{\vec{R}} = m_1\dot{\vec{r}}_1 + m_2\dot{\vec{r}}_2$, where $M = m_1 + m_2$ is the total mass, and find the general solution for $\vec{R}(t)$ (2 points).
3. Use the special solution $\vec{R} = (m_1\vec{r}_1 + m_2\vec{r}_2)/M$, to express $\vec{r}_1$ and $\vec{r}_2$ through $\vec{R}$ and $\vec{r}$ (3 points).

   The center of mass (cm) systems is defined by $\vec{R}(t) = 0$. Show the following equalities in the system .
4. $T_{\rm cm} = m_1\, \vec{v}_1^{\,2}/2 + m_2\, \vec{v}_2^{\,2}/2 = \mu\, \vec{v}^{\,2}/2$ with $\vec{v} = \dot{\vec{r}}$ and μ the reduced mass. Express μ through m_1, m_2 and M (3 points).
5. $\vec{L} = \vec{r}_1 \times \vec{p}_1 + \vec{r}_2 \times \vec{p}_2 = \mu\, \vec{r} \times \vec{v}$ (2 points).