ADVANCED DYNAMICS — PHY 4241/5227

HOME AND CLASS WORK – SET 7

(February 15, 2009)

Read M&T chapter 8.1 to 8.7 (due February 16).

(26) Assume a central potential and $\vec{L} \neq 0$.

1. Use initial conditions in the CM system, $\vec{r}_0$ and $\vec{v}_0$ to define unit vectors $\hat{e}_1$, $\hat{e}_2$ and $\hat{e}_3$ of a Cartesian coordinates system in which
$$L = |\vec{L}| = \mu\, r^2\, \dot{\phi}\,, \quad \phi_0 = 0\,,\ \dot{\phi}_0 > 0$$
holds (ϕ azimuth angle). Due February 16 in class (3 points).
2. Express the total energy $E_{\rm cm}$ in terms of r and ϕ and their derivatives (i.e., get rid of $\vec{v}$). Substitute the conserved angular momentum L into your expression. Combine all terms, which contain r but not $\dot{r}$, into the definition of an effective potential $U_{\rm eff}(r)$. Due February 16 in class (5 points).

Turn in both February 18 before class.

(27) Use (in arbitrary units) the parameters $\alpha = \mu = L = 1$ to plot the potential and the effective potential of the Kepler problem
$$U = -\frac{\alpha}{r} \quad \text{and} \quad U_{\rm eff}(r) = U(r) + \frac{L^2}{2\,\mu\, r^2}$$
in the range $0 < r \leq 10$ with the ordinate restricted to the interval $[-1, 0.6]$. Describe the motion for $E_{\rm cm} = U_{\rm eff}^{\rm min}$, where $U_{\rm eff}^{\rm min} = U_{\rm eff}(r^{\rm min})$ is the minimum of the effective potential. Calculate $r^{\rm min}$ as function of L, μ and α and give then the numerical value. Due February 18 before class (8 points).

(28) Separate variables in the equation
$$E_{\rm cm} = \frac{\mu}{2}\dot{r}^2 + U_{\rm eff}(r)$$
to derive an integral equation for $t(r)$. Due February 18 in class (4 points).

(29) Use angular momentum conservation and the separation of variables in the previous problem to derive an integral equation for $\phi(r)$. Due February 18 in class (4 points).

(30) Kepler problem: Use unit with $G = 1$ for the gravitational constant and the initial conditions given in the table on the next page to plot the four orbits in the coordinate system used in chapter 8 of T&M (put them all in one plot). If you want, you can download the data in the file `Korbits.txt`. Due February 23 before class (10 points homework).

(31) Get the initial positions into the plot of the previous homework. March 2 before class (4 extra points).

(32) Find positions and velocities in the original system at time $t = 1.5$ (arbitrary units and $G = 1$). Ten extra points when turned in by March 16.

(33) Assume $0 < e < 1$ for the eccentricity and transform the elliptic equation

$$\frac{r}{p} = 1 - e\,\cos(\phi)\,, \quad p > 0$$

into the form

$$\frac{x'^2}{a^2} + \frac{y'2}{b^2} = 1\ .$$

This means, *derive* the definitions of x', y', major half-axis a and minor half-axis b in terms of x, y, p and e (p and e are Landau-Lifshitz notation in M&T it is α and ϵ). Due February 20 in class (5 points).

	Masses		Initial Positions			Initial Velocities		
#	i	m_i	$x^1_{i,0}$	$x^2_{i,0}$	$x^3_{i,0}$	$\dot{x}^1_{i,0}$	$\dot{x}^2_{i,0}$	$\dot{x}^3_{i,0}$
1	1	0.651	0.585	-0.238	-0.755	-0.828	-0.865	-0.726
	2	0.931	-0.096	0.000	0.357	-0.209	0.107	-0.660
2	1	1.510	0.460	-0.359	-0.234	-0.918	-0.941	-0.323
	2	0.126	-0.066	-0.090	-0.809	0.789	0.788	0.620
3	1	1.328	-0.125	0.898	0.194	-0.452	0.172	0.125
	2	1.999	-0.449	-0.085	-0.454	-0.976	-0.990	-0.968
4	1	0.180	0.204	-0.968	-0.753	-0.811	-0.632	0.784
	2	1.560	-0.889	-0.979	0.854	-0.323	-0.774	-0.533

Table 1: Initial conditions for the Kepler problem (arbitrary units and $G = 1$).