ADVANCED DYNAMICS — PHY 4241/5227
HOME AND CLASS WORK – SET 10

(March 2, 2009)

(41)

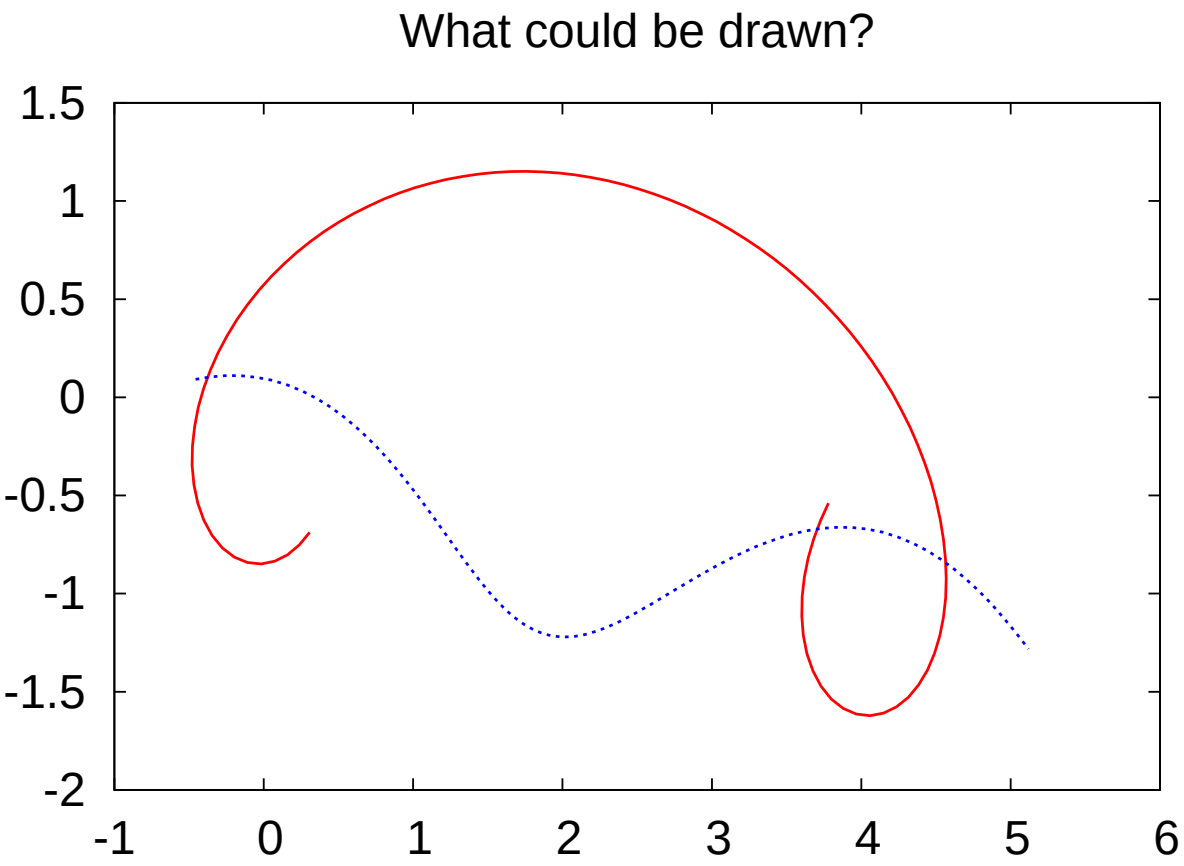
1. Show $1/\sqrt{1-\beta^2} = \cosh(\zeta)$. Due March 2 in class (2 points).
2. Show $\cosh(\zeta_2)\,\cosh(\zeta_1) + \sinh(\zeta_2)\,\sinh(\zeta_1) = \cosh(\zeta_1+\zeta_2)$. Due March 2 in class (2 points).
3. Velocities are given by $\beta_1 = \tanh(\zeta_1)$ and $\beta_2 = \tanh(\zeta_2)$. Use the addition theorem of rapidities to express the velocity given by $\beta = \tanh(\zeta_1+\zeta_2)$ in terms of β_1 and β_2. Due March 2 in class (4 points).

(42) D mesons have a (mean) lifetime of 1051×10^{-15} s. Assume such a meson is created in a particle collission at Fermilab and moves at 0.995 c with respect to the Lab frame. Compute the (mean) distance travelled by such a D meson. Due March 6 before class (6 points).

(43) Assume that a rocket ship leaves the earth in the year 2020. The rocket ship is so constructed so that it has an acceleration g in its own frame (to make the occupants feel comfortable). It accelerates on a straight-line path for 5 years (by its own clocks), decelerates at the same rate for 5 more years, turns around, accelerates for 5 years, decelerates for 5 years, and lands on earth. What year is on earth? Use $g = 9.81\ [m/s^2]$, one year $= 365 \times 24 \times 3600\ [s]$, and the speed of light c, and assume that earth defines an inertial system. Due March 16 before class (10 points).

(44)

1. Write down (again) the basic postulates of the Special Theory of Relativity. Due March 16 in class (2 points).
2. Let $S_{\alpha\beta}$ be a (covariant) symmetric tensor. Write $S_{\alpha\beta}\,dx^\alpha dx^\beta$ explicitly as a sum of ten terms. Due March 20 in class (2 points).
3. Let $F_{\alpha\beta}$ be a (covariant) antisymmetric tensor. Calculate $F_{\alpha\beta}\,dx^\alpha dx^\beta$ (write it as a sum of six terms. Due March 20 in class (2 points).
4. Calculate $h^\alpha{}_\beta = g^{\alpha\gamma}g_{\gamma\beta}$ and $h_\alpha{}^\beta = g_{\alpha\gamma}g^{\gamma\beta}$. (A) As g-tensor. (B) The explicit matrix and identify then with another well-known symbol. Due March 20 in class (2 points).
5. Write down explicitly the matrix $(a^\beta{}_\alpha)$ and its transpose $(\tilde{a}^\alpha_\beta)$, $\tilde{a}_\beta{}^\alpha = a^\beta{}_\alpha$. Due March 23 in class (2 points).
6. 2 points for the first correct guess about the figure next page. Done (went to Evan).



(45) Continuation of (43). Seen from earth, how far away did the spaceship travel? Express the result in light years. Due March 20 before class (6 points).

(46) Another continuation of (43). Assume that the spaceship moves by transforming mass into light, which is exhausted.

1. Derive an expression for $m(\tau)$, the (remaining) mass of the spaceship at proper time τ. (Hint: Use momentum conservation in the instanteneous rest frame.)
2. Which fraction of the original mass is left, after the spacetrip has been performed?

Due March 27 before class (8 points).