ADVANCED DYNAMICS — PHY 4241/5227

HOME AND CLASS WORK – SET 11

(March 23, 2009)

(47) The matrix L is defined by

$$L = \begin{pmatrix} l^0{}_0 & l^0{}_1 & l^0{}_2 & l^0{}_3 \\ l^1{}_0 & l^1{}_1 & l^1{}_2 & l^1{}_3 \\ l^2{}_0 & l^2{}_1 & l^2{}_2 & l^2{}_3 \\ l^3{}_0 & l^3{}_1 & l^3{}_2 & l^3{}_3 \end{pmatrix}$$

1. Calculate $-g\,L$ (2 points).
2. Write down the transpose matrix $\tilde{L}$ (2 points).
3. Calculate $\tilde{L}\,g$ (2 points).
4. Compare 1. and 2. to find the general form of L (*i.e.* use $\tilde{L}\,g = -g\,L$, 4 points).
5. Obtain the same result by discussing the elements of the equation $g^{\alpha\beta}\tilde{l}_\beta{}^\gamma g_{\gamma\delta} = -l^\alpha{}_\delta$ (4 extra points. Hint: Do the contractions first.)

Due March 25 in class.

(48) In the following K_1 and S_3 are generators as defined in class.

1. Write down the Taylor expansions of the functions $\cosh(x)$, $\cos(x)$, $\sinh(x)$ and $\sin(x)$ (4 points).
2. Calculate $\exp(-\zeta\,K_1)$ and explain its physical meaning (3 points).
3. Calculate $\exp(-\omega\,S_3)$ and explain its physical meaning (3 points).

Due March 27 in class.

(49) Consider the spaceship journey a last time and plot versus the proper time the following quantities (due March 30 before class, 10 points):

1. The time on earth at which the spaceship, seen from earth, is at the particular position.
2. The distance from earth as seen from earth.
3. The date of news received from earth, which is transmitted at the speed of light.

(50) Derive the Taylor expansions of the functions $\exp(x)$, $\cosh(x)$, $\cos(x)$, $\sinh(x)$ and $\sin(x)$ about $x = 0$ from the equation

$$f(x) = \sum_{n=0}^{\infty} f^{(n)}(0)\,\frac{x^n}{n!}$$

by calculating $f^{(n)}(0)$ for each case explicitly. Do NOT use Euler's formula. Due April 1 before class (5 points).