ADVANCED DYNAMICS — PHY 4241/5227

HOME AND CLASS WORK – SET 12

(March 31, 2009)

(51) For light from some galaxy the Balmer spectrum of the hydrogen atom is found at wavelengths $\lambda = (729.2\,\mathrm{nm})\, m^2/(m^2-4)$, $m = 3, 4, 5, \ldots$. Neglect expansion of space and find the speed at which the galaxy moves away or towards us. Due April 1 before class (4 points).

(52) (1) Taylor expand $\sqrt{x+\epsilon}$ to leading order in ϵ. (2) Substitute $x = c^4 m_0^2$ and $\epsilon = c^2\vec{p}^2$. (3) Compare with $mv^2/2$. Due April 1 in class (4 points).

(53) Derive for the momentum in the rest frame $p^0 = c\, m_0$, where m_0 is the rest mass, from the relativistic transformation laws and the known definition of momentum in the non-relativistic limit. Due April 1 in class (4 points).

(54) (1) Write down the definition of ∂_α. Due April 3 in class (1 point).

(2) Let A^α be a vector field. Write down the scalar form this and ∂_α (no scalar operators). Due April 3 in class (1 point).

(3) Use the quantities of assignments (1) and (2) to write down a rank two tensor $T^{\alpha\beta}$ field (no tensor operator). Due April 3 in class (1 point).

(4) Write the equation

$$\partial_\alpha\, F^{\alpha\beta} = \frac{4\pi}{c}\, J^\beta$$

explicitly out for $\beta = 0, 1, 2, 3$. Due April 6 in class (4 points).

(5) Compare the $\beta = 0$ result with $\nabla\vec{E} = 4\pi\rho$ and the $\beta = i = 1, 2, 3$ results with $\nabla \times \vec{B} - c^{-1}\left(\partial\vec{E}/\partial t\right) = 4\pi\vec{J}/c$. Due April 6 in class (4 points).

(55) An electron and a positron, each with mass equal to 0.511 MeV, annihilate at rest into two photons. Chose the rest frame of the positronium, such that one of the photons moves in positive $x = x^1$ direction.

1. For each photon find the momentum 4-vector.
2. Find the 4-momentum in a frame that moves at a velocity $c\beta\hat{x}$ with respect to the rest frame of the positronium.
3. Suppose that the annihilation took place 10^9 years ago in a galaxy that is receding from us at $\beta = 4/5$. What is the energy of the photon that we observe?

Due April 6 before class (10 points).