(56) Let $T^{\alpha\beta}$ be a rank two tensor.

1. Write down $T^{\alpha\beta}_S$, the symmetric part of $T^{\alpha\beta}$. Due April 8 in class (1 point).
2. Write down $T^{\alpha\beta}_A$, the antisymmetric part of $T^{\alpha\beta}$. Due April 8 in class (1 point).
3. Calculate $T^{\alpha\beta}_A T^{\beta\alpha}_S$. Due April 8 in class (1 point).
4. Derive the continuity equation from the antisymmetry of $F^{\alpha\beta}$. Due April 8 in class (1 point).
5. Show $\partial_\alpha \ast F^{\alpha\beta} = 0$, where

$$\ast F^{\alpha\beta} = \frac{1}{2} \varepsilon^{\alpha\beta\gamma\delta} F_{\gamma\delta}$$

is the dual tensor. Here $\varepsilon^{\alpha\beta\gamma\delta}$ is the completely antisymmetric Levi-Cevita tensors: $+1$ for $(\alpha, \beta, \gamma, \delta)$ and even permutation of $(0, 1, 2, 3)$, $-1$ for $(\alpha, \beta, \gamma, \delta)$ and even permutation of $(0, 1, 2, 3)$. Due April 8 in class (2 points).

(57)

1. Write down $(F_{\alpha\beta})$ in $\vec{E}$ and $\vec{B}$ fields starting from the corresponding results for $(F^{\alpha\beta})$. Due April 10 in class (3 points).
2. What is the value of $\epsilon_{0123}$? Due April 10 in class (1 point).
3. Write down $(\ast F^{\alpha\beta})$ in $(F_{\alpha\beta})$ fields. Due April 10 in class (3 points).
4. Write down $(\ast F^{\alpha\beta})$ in $(F^{\alpha\beta})$ fields. Due April 10 in class (3 points).
5. Write down $(\ast F^{\alpha\beta})$ in $\vec{E}$ and $\vec{B}$ fields. Due April 10 in class (3 points).
6. Write down $0 = \partial_\alpha \ast F^{\alpha0}$ in $\vec{E}$ and $\vec{B}$ fields. Due April 10 in class (1 point).
7. Write down $0 = \partial_\alpha \ast F^{\alpha1}$ in $\vec{E}$ and $\vec{B}$ fields. Due April 10 in class (1 point).
8. Under a gauge transformation the vector potential transform according to

$$A^\alpha \rightarrow A'^\alpha = A^\alpha + \partial^\alpha \Lambda.$$ 

Calculate the corresponding transformation of the electromagnetic field tensor

$$F^{\alpha\beta} \rightarrow F'^{\alpha\beta}.$$ 

Due April 10 in class (3 points).