ADVANCED DYNAMICS — PHY 4241/5227

HOME AND CLASS WORK – SET 14

(April 13, 2009)

(58) The electromagnetic field tensor transforms according to

$$F'^{\alpha\beta} = a^{\alpha}_{\ \gamma} a^{\beta}_{\ \delta} F^{\gamma\delta} .$$

1. Choose the particular case of a Lorentz boost in x^1-direction, $\vec{v} = v\,\hat{e}_1$, and write down the transformation law for the electric field $\vec{E}$ and the magnetic induction $\vec{B}$. Due April 15 before class (8 points).
2. Compare with the results you get from (due April 15 before class (4 points).

$$\begin{aligned} \vec{E}' &= \gamma\left(\vec{E} + \vec{\beta} \times \vec{B}\right) - \frac{\gamma^2}{\gamma+1}\vec{\beta}\left(\vec{\beta}\,\vec{E}\right) , \\ \vec{B}' &= \gamma\left(\vec{B} - \vec{\beta} \times \vec{E}\right) - \frac{\gamma^2}{\gamma+1}\vec{\beta}\left(\vec{\beta}\,\vec{B}\right) . \end{aligned}$$

(59) In the same way that the contraction, or relativistic dot product, of two four vectors is a Lorentz invariant, the contraction of two relativistic tensors is also a Lorentz invariant (Griffiths Problem 12.50).

1. Compute the three Lorentz invariants from the contraction of the tensors:

$$F^{\mu\nu} F_{\mu\nu} \ , \ \ {}^*F^{\mu\nu}\, {}^*F_{\mu\nu} \text{ and } F^{\mu\nu}\, {}^*F_{\mu\nu}$$

   in terms of the electric and magnetic fields $\vec{E}$ and $\vec{B}$. Due April 15 in class (3 points).

2. Suppose that in one inertial frame $\vec{B}=0$ but $\vec{E}\neq 0$ (at some point P). Is it possible to find another system in which the *electric field* is zero at P? Due April 15 in class (1 point).

(60) The non–zero fields $\vec{E}$ and $\vec{B}$ are non–parallel in inertial frame K. Inertial frame K' moves with velocity $\vec{v}$ with respect to K. Find a physical velocity $\vec{v}$ so that $\vec{E}'$ and $\vec{B}'$ are parallel. (Hint: Try $\vec{E} = E_2\,\hat{e}_2$, $\vec{B} = B_2\,\hat{e}_2 + B_3\,\hat{e}_3$ and $\vec{v} = v\,\hat{e}_1$.) Due April 17 before class (10 points).

(61) Transform

$$f^i = \frac{q}{c}\, F^{i\beta}\, U_\beta$$

into

$$\vec{f} = q\,\gamma\,\vec{E} + \frac{q}{c}\,\vec{U} \times \vec{B} \ .$$

Due April 17 in class (4 points).