The electromagnetic field tensor transforms according to

\[ F^\alpha_\beta = a^\alpha_\gamma a^\beta_\delta F^\gamma_\delta. \]

1. Choose the particular case of a Lorentz boost in \( x^1 \)-direction, \( \vec{v} = v \hat{e}_1 \), and write down the transformation law for the electric field \( \vec{E} \) and the magnetic induction \( \vec{B} \). Due April 15 before class (8 points).

2. Compare with the results you get from (due April 15 before class (4 points).

\[
\begin{align*}
\vec{E}' & = \gamma (\vec{E} + \vec{B} \times \vec{v}) - \frac{\gamma^2}{\gamma + 1} \vec{\beta} (\vec{\beta} \vec{E}) , \\
\vec{B}' & = \gamma (\vec{B} - \vec{B} \times \vec{E}) - \frac{\gamma^2}{\gamma + 1} \vec{\beta} (\vec{\beta} \vec{B}) .
\end{align*}
\]

(59) In the same way that the contraction, or relativistic dot product, of two four vectors is a Lorentz invariant, the contraction of two relativistic tensors is also a Lorentz invariant (Griffiths Problem 12.50).

1. Compute the three Lorentz invariants from the contraction of the tensors:

\[ F^{\mu\nu} F_{\mu\nu}, \quad *F^{\mu\nu} *F_{\mu\nu} \quad \text{and} \quad F^{\mu\nu} *F_{\mu\nu} \]

in terms of the electric and magnetic fields \( \vec{E} \) and \( \vec{B} \). Due April 15 in class (3 points).

2. Suppose that in one inertial frame \( \vec{B} = 0 \) but \( \vec{E} \neq 0 \) (at some point \( P \)). Is it possible to find another system in which the electric field is zero at \( P \)? Due April 15 in class (1 point).

(60) The non–zero fields \( \vec{E} \) and \( \vec{B} \) are non–parallel in inertial frame \( K \). Inertial frame \( K' \) moves with velocity \( \vec{v} \) with respect to \( K \). Find a physical velocity \( \vec{v} \) so that \( \vec{E}' \) and \( \vec{B}' \) are parallel. (Hint: Try \( \vec{E} = E_2 \hat{e}_2, \vec{B} = B_2 \hat{e}_2 + B_3 \hat{e}_3 \) and \( \vec{v} = v \hat{e}_1 \).) Due April 17 before class (10 points).

(61) Transform

\[ f^i = \frac{q}{c} F^{i\beta} U_\beta \]

into

\[ \vec{f} = q \gamma \vec{E} + \frac{q}{c} \vec{U} \times \vec{B} . \]

Due April 17 in class (4 points).