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FIRST MIDTERM EXAM

PROBLEM 1

FROM ALL POSSIBLE PATHS THAT A PHYSICAL SYSTEM CAN TAKE IN GOING FROM POINT 1 AT TIME t_1 TO POINT 2 AT TIME t_2 IT WILL SELECT THE ONE THAT MINIMIZES (ACTUALLY EXTREMISES) THE ACTION. FOR

$$L = \frac{1}{2} m\dot{x}^2 - V(x)$$

THE EULER-LAGRANGE EQUATION OF MOTION IS GIVEN BY

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} = 0 \Rightarrow \boxed{m\ddot{x} = -V'(x) = F(x)}$$

IN COMPLETE AGREEMENT WITH NEWTON'S SECOND LAW.

PROBLEM 2

A LAGRANGIAN DEPENDS IN GENERAL ON ALL GENERALIZED COORDINATES, ALL GENERALIZED VELOCITIES, AND TIME. THAT IS,

$$L = L(q_i; \dot{q}_i; t).$$

THE GIVEN CENTRAL FORCE FIELD LAGRANGIAN IS INDEPENDENT OF ϕ AND TIME t. THUS,

a) $\dfrac{\partial L}{\partial t} = 0 \Rightarrow E = T + V$ (WHICH EQUALS THE HAMILTONIAN FUNCTION) IS CONSERVED

b) $\dfrac{\partial L}{\partial \phi} = 0 \Rightarrow p_\phi = \dfrac{\partial L}{\partial \dot{\phi}} = mr^2 \sin^2\theta\, \dot{\phi}$ IS CONSERVED.

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PROBLEM 3

THE POISSON BRACKET OF $f = x^2$ WITH $H = \frac{p^2}{2} + \frac{x^2}{2}$ IS GIVEN BY,

$$[f,H] = \frac{\partial f}{\partial x}\frac{\partial H}{\partial p} - \cancelto{0}{\frac{\partial f}{\partial p}}\frac{\partial H}{\partial x} = (2x)(p) = 2xp$$

SINCE

$$\frac{df}{dt} = [f,H] + \frac{\partial f}{\partial t} = 2xp \neq 0$$

f IS NOT A CONSTANT OF THE MOTION.

PROBLEM 4

MAXWELL'S EQUATIONS IN GAUSSIAN UNITS ARE GIVEN BY

$$\boxed{\begin{array}{ll} \nabla\cdot\vec{E} = 4\pi\rho & \nabla\cdot\vec{B} = 0 \\ \nabla\times\vec{E} + \frac{1}{c}\frac{\partial\vec{B}}{\partial t} = 0 & \nabla\times\vec{B} - \frac{1}{c}\frac{\partial\vec{E}}{\partial t} = \frac{4\pi}{c}\vec{J} \end{array}}$$

PROBLEM 5

BEFORE MAXWELL'S CONTRIBUTION AMPERE'S LAW (IN GAUSSIAN UNITS) WAS GIVEN BY: $\nabla\times\vec{B} = \frac{4\pi}{c}\vec{J}$. MAXWELL REALIZED THAT THE CONTINUITY EQUATION FOR ELECTRIC CHARGE $\nabla\cdot\vec{J} + \frac{\partial\rho}{\partial t} = 0$ WAS NOT CONSISTENT WITH AMPERE'S LAW SINCE:

$$\nabla\cdot(\nabla\times\vec{B}) = 0 = \frac{4\pi}{c}\nabla\cdot\vec{J} \Rightarrow \nabla\cdot\vec{J} = 0$$

THUS, HE INTRODUCED THE TERM $\frac{1}{c}\frac{\partial\vec{E}}{\partial t}$ (SEE PROBLEM 4)

THEREBY RESTORING CURRENT CONSERVATION AND UNIFYING ELECTRODYNAMICS WITH OPTICS.

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PROBLEM 5

a) $\vec{E} = \frac{\alpha}{r^2}\hat{r}$ AND $\vec{B}=0$ IS POSSIBLE.

THIS IS THE CONFIGURATION OF FIELDS OBTAINED BY HAVING A (STATIC) CHARGE OF MAGNITUDE α LOCATED AT THE ORIGIN.

b) $\vec{B} = \frac{\alpha}{r^2}\hat{r}$ AND $\vec{E}=0$ IS IMPOSSIBLE.

THIS IS THE CONFIGURATION OF FIELDS OBTAINED BY HAVING A (STATIC) MAGNETIC MONOPOLE OF MAGNITUDE α LOCATED AT THE ORIGIN. THAT IS, FOR THIS CONFIGURATION OF FIELDS $\nabla\cdot\vec{B}\neq 0$. ($\vec{E}=0$ IS IN GENERAL OK).