

# ADVANCED MECHANICS — PHY-4241/5227

## HOMEWORK 1

(January 5, 2004)

Due on Monday, January 12, 2004

### PROBLEM 1

Read, and read again, “**The Principle of Least Action**” from Chapter 19 of “*The Feynman Lectures on Physics*”, Vol. II.

### PROBLEM 2

Problem 6-7 of T& M.

### PROBLEM 3

Consider a particle of mass  $m \equiv 1$  moving, from  $x_1 = 0$  at time  $t_1 = 0$  to  $x_2 = 1$  at time  $t_2 = \pi/2$ , under the influence of a one-dimensional harmonic potential of the form:

$$V(x) = \frac{1}{2}x^2.$$

- a) Using Euler-Lagrange’s equations of motion, obtain the time-dependent motion of the system; *i.e.*, solve for  $x(t)$ . Compute the action for this exact path.
- b) Using an approximate linear path of the form  $x(t) = a + bt$ , compute the action for this path and compare it with the exact value obtained in part a).
- c) Using an approximate quadratic path of the form  $x(t) = a + bt + ct^2$ , compute the action for this path and compare it with the exact value obtained in part a).

**Hint:** For parts b) and c) make sure that the paths are consistent with the boundary conditions. If any constant remains undetermined, fix it by minimizing the action.