ADVANCED DYNAMICS — PHY–4241/5227

HOMEWORK 5

(February 2, 2004)
Due Monday, February 9, 2004 (late afternoon)

PROBLEM 12

Prepare to present Chapter 8, up to equation (8.17), of Marion and Thornton (4th edition) in class.

PROBLEM 13

The Lorentz Force $\vec{F} = q\,(\vec{E} + c^{-1}\vec{v} \times \vec{B})$ acts on a point particle of mass m and charge q. Consider the case of crossed, constant magnetic and electic fields $\vec{B} = (0, 0, B_z)$ and $\vec{E} = (E_x, E_y, 0)$.

a) Find and discuss the general solution of the equation of motion. Hint: Introduce complex quantities $\zeta = x + iy$ and $\varepsilon = E_x + iE_y$ and solve the linear differential equation of ζ with complex constants.

b) Eliminate the integration constants through the initial conditions $\vec{r}(0) = \vec{r}_0$ and $\vec{v}(0) = \vec{v}_0$ and perform the limit $B_z \to 0$ in the equations of motion.

PROBLEM 14

The Lagrangian for a particle of mass m and electric charge q moving under the influence of a magnetic (but no electric) field is given by

$$L(\mathbf{r}, \dot{\mathbf{r}}) = \frac{1}{2}m\dot{\mathbf{r}}^2 + \frac{q}{c}\dot{\mathbf{r}} \cdot \mathbf{A}(\mathbf{r}) \, ,$$

where $\mathbf{A}(\mathbf{r})$ is the vector potential. Assume that the magnetic field is constant and given by $\mathbf{B}(\mathbf{r}) = B_0\hat{\mathbf{z}}$.

a) Show that for such a constant magnetic field the vector potential can be written in the form $\mathbf{A}(\mathbf{r}) = \frac{1}{2}(\mathbf{B} \times \mathbf{r})$. That is, show that such a vector potential satisfies: $\nabla \times \mathbf{A} = \mathbf{B}$.

b) Construct the Hamiltonian of the system in terms of the Cartesian coordinates and the corresponding canonical momenta of the particle.

c) Denoting by $\boldsymbol{\pi} \equiv m\dot{\mathbf{r}} = m\mathbf{v}$ the "mechanical" momentum of the particle, evaluate the following Poisson brackets:

$$[\pi_x, \pi_y] \, , \; [\pi_y, \pi_z] \, , \; [\pi_z, \pi_x] \, .$$

Turn over to the second page!

d) By re-expressing the Hamiltonian of part b) in terms of the mechanical momentum of the particle—and by using the results derived in part c)—obtain the most general solution for $\boldsymbol{\pi}(t)$ by using Poisson's equation:

$$\frac{d\boldsymbol{\pi}}{dt} = [\boldsymbol{\pi}, H] \ .$$

Interpret your results on the basis of Newton's second law plus Lorentz force.