ADVANCED DYNAMICS — PHY–4241/5227

HOMEWORK 13

(Monday, April 12, 2004)

Due on Monday (afternoon), April 18, 2004

PROBLEM 30

(Problem 9.9 Griffiths)

Write down the (real) electric and magnetic fields for a monochromatic plane wave of amplitude E_0, frequency ω, and linear polarization $\hat{\mathbf{n}}$ that is:

a) Traveling in the negative x direction and polarized in the z direction.

b) Traveling in the direction from the origin to the point $(1,1,1)$ with polarization parallel to the xz plane.

In each case sketch the wave and give the explicit Cartesian components of $\hat{\mathbf{k}}$ and $\hat{\mathbf{n}}$.

PROBLEM 31

Consider a linear medium of dielectric constant ϵ and permeability μ. Further, assume that the energy density (*i.e.,* the energy per unit volume) stored in space is given by:

$$u = \frac{1}{8\pi}\left(\mathbf{E}\cdot\mathbf{D} + \mathbf{B}\cdot\mathbf{H}\right) \ .$$

a) Starting from the above equation, and using Maxwell's equations, show that the energy satisfies a continuity like equation of the form:

$$\frac{\partial u}{\partial t} + \nabla\cdot\mathbf{S} = -\mathbf{J}\cdot\mathbf{E} \ .$$

Display an explicit form for the *Poynting* vector $\mathbf{S}$ in terms of the electromagnetic fields. What is the physical significance of the $\mathbf{J}\cdot\mathbf{E}$ term?

b) Compute the *time-averaged* Poynting vector for an electromagnetic plane wave of wave number $\mathbf{k}$ and frequency ω. Note that the definition of the time-averaged Poynting vector is given by

$$\langle\mathbf{S}\rangle_{\rm avg} = \frac{1}{T}\int_0^T \mathbf{S}(\mathbf{x},t)dt \ , \qquad (T = 2\pi/\omega) \ .$$

Turn over!

PROBLEM 32

An electromagnetic field (in vacuum) is given by the real part of

$$\vec{E} = \vec{E}_0 e^{i\vec{k}\vec{x}-i\omega t} \quad (E_0 \text{ real}) \quad \text{and} \quad \vec{B} = \frac{\vec{k} \times \vec{E}}{|\vec{k}|}, \quad (\vec{k} \text{ real}).$$

(a) Use the physical fields to calculate the energy density and the Poynting vector (real definitions) pointwise as well as their time averages.

(b) Compare the Poynting vector of (a) with the complex definition

$$\vec{S} = \frac{c}{8\pi}\,\vec{E} \times \vec{B}^* \quad (\vec{k} \text{ still real}).$$

In what sense are identical results obtained?

(c) Assume perfect absorption, what is the pressure on a surface perpendicular to $\vec{k}$?

Fini!