ADVANCED DYNAMICS — PHY–4241/5227

HOMEWORK 3

(January 18, 2003)
Due on Monday, January 27, 2003

PROBLEM 7

(Example 7-12 Marion and Thornton)

Consider a spherical (*i.e.,* three-dimensional) pendulum of mass m and length b, as displayed in Figure 7-10 of Marion and Thornton.

a) Using spherical coordinates, obtain the Lagrangian of the system $L(\theta, \phi, \dot{\theta}, \dot{\phi})$.

b) Using the Legendre transformation

$$H(\theta, \phi, p_\theta, p_\phi) = \dot{\theta} p_\theta + \dot{\phi} p_\phi - L(\theta, \phi, \dot{\theta}, \dot{\phi}) ,$$

construct the Hamiltonian of the system and show that it is identical to the energy $E = T + V$.

c) Using the method of the effective potential, obtain the conditions for which the pendulum will move in an orbit of constant height, that is, of constant $\theta = \theta_0$.

PROBLEM 8

(Problem 7-30 Marion and Thornton)

Consider any two continuous functions of the generalized coordinates and momenta (and possible time) $g(q_k, p_k, t)$ and $h(q_k, p_k, t)$. The **Poisson bracket** of g and h is defined by

$$[g, h] \equiv \sum_k \left(\frac{\partial g}{\partial q_k} \frac{\partial h}{\partial p_k} - \frac{\partial g}{\partial p_k} \frac{\partial h}{\partial q_k} \right) .$$

Verify the following properties of the Poisson brackets:

a) $\dfrac{dg}{dt} = [g, H] + \dfrac{\partial g}{\partial t}$.

b) $\dot{q}_j = [q_j, H]$; $\quad \dot{p}_j = [p_j, H]$.

c) $[q_i, q_j] = [p_i, p_j] = 0$; $\quad [q_i, p_j] = \delta_{ij}$.

PROBLEM 9

a) Show that if f, g, and h are arbitrary functions of q and p (and possible time) then the following relations among Poisson brackets hold true:

$$\begin{aligned} [fg,h] &= [f,h]g + f[g,h] \ , \\ [f,gh] &= [f,g]h + g[f,h] \ . \end{aligned}$$

b) Show that the Poisson bracket between an **arbitrary** function of $r = \sqrt{(x^2+y^2+z^2)}$ and the momentum $\mathbf{p}$ is given by

$$[f(r), p_i] = \frac{x_i}{r} f'(r) \ .$$

c) Assuming the validity of the fundamental Poisson-bracket relations between coordinates and momenta

$$[x_i, x_j] = [p_i, p_j] = 0 \ ; \quad [x_i, p_j] = \delta_{ij} \ ,$$

compute the following Poisson brackets:

$$[x_i, L_j] \ , \ [p_i, L_j] \ , \text{ and } \ [L_i, L_j] \ ,$$

where $\mathbf{L} = \mathbf{r} \times \mathbf{p}$ is the angular momentum of the system.