ADVANCED DYNAMICS — PHY–4241/5227

HOMEWORK 8 and 9

(March 1, 2003)

Due on Monday, March 17, 2003

PROBLEM 23

(Problem 12.13 Griffiths)

Sophie Zabar, clairvoyant, cried out in pain at precisely the instant that her twin brother, 500 km away, hit his thumb with a hammer. A skeptical scientist observed both events (brother's accident, Sophie's cry) from an airplaine traveling at 12c/13 to the right (see Fig. 12.19). Which event occurred first according to the scientist? How much earlier was it, in seconds?

PROBLEM 24

(Problem 12.18 Griffiths)

(a) Write out the matrix that describes a Galilean transformation (Eq. 12.12).

(b) Write out the matrix that describing a Lorentz transformation along the y axis.

(c) Find the matrix describing the Lorentz transformation with velocity v along the x axis followed by a Lorentz transformation with velocity $\bar{v}$ along the y axis. Does it matter in what order the transformations are carried out?

PROBLEM 25

(Problem 12.20 Griffiths)

(a) Event A happens at point $(x_A = 5, y_A = 3, z_A = 0)$ and at a time t_A given by $ct_A = 15$; event B occurs at $(10, 8, 0)$ and $ct_B = 5$, both in system $\mathcal{S}$.

(i) What is the invariant interval between A and B?

(ii) Is there an inertial frame in which they occur simultaneously? If so, find its velocity (magnitude and direction) relative to $\mathcal{S}$.

(ii) Is there an inertial frame in which they occur at the same point? If so, find its velocity (magnitude and direction) relative to $\mathcal{S}$.

(b) Repeat part (a) for $A = (2, 0, 0)$, $ct_A = 1$; and $B = (5, 0, 0)$, $ct_B = 3$.

PROBLEM 26

An electron in its own rest frame has an energy equal to $E = m_0c^2$ and zero linear momentum $\mathbf{p} = 0$. Compute the energy and momentum of the electron as measured by an observer in the *"laboratory"* frame. Note that in the laboratory frame the electron is moving

with constant linear velocity v along the x-axis; that is, $\mathbf{v} = v\hat{\mathbf{x}}$. Show that while E and p are different in both frames, the quantity $E^2 - (pc)^2$ is invariant.

Hint: Under a Lorentz transformation the energy (momentum) transforms as the timelike (spacelike) component of a four-vector.

PROBLEM 27

Two electrons are moving toward each other. As measured in the laboratory frame electron 1 has a speed of $v_1 = \beta_1 c$ and electron 2 of $v_2 = \beta_2 c$. What is the speed of electron 1 in the rest frame of electron 2?

PROBLEM 28

Cosmic rays are very energetically particles (such as protons) that occasionally collide with the Earth's atmosphere producing a copious amount of pions—strongly interacting particles with a finite lifetime. For example, charged pions, which decay via the weak interaction into muons and neutrinos, have a lifetime of $\tau = 2.6 \times 10^{-8}$ s. Compute the total distance traveled by a pion moving at 99% of the speed of light as measured by a *"terrestrial"* observer.