

Solution for the $[L_i, L_j]$ part of assignment 24.5 set 6.

$$\begin{aligned}
[L_i, L_j] &= \epsilon_{ikl}\epsilon_{jmn} \left(\frac{\partial(x_k p_l)}{\partial x_r} \frac{\partial(x_m p_n)}{\partial p_r} - \frac{\partial(x_m p_n)}{\partial x_r} \frac{\partial(x_k p_l)}{\partial p_r} \right) \\
&= \epsilon_{ikl}\epsilon_{jmn} (\delta_{kr}\delta_{nr}x_m p_l - \delta_{mr}\delta_{lr}x_k p_n) = \epsilon_{ikl}\epsilon_{jmn} (\delta_{kn}x_m p_l - \delta_{ml}x_k p_n) \\
&= \epsilon_{ikl}\epsilon_{jmk}x_m p_l - \epsilon_{ikl}\epsilon_{jln}x_k p_n = \epsilon_{kli}\epsilon_{kjm}x_m p_l - \epsilon_{lik}\epsilon_{lnj}x_k p_n \\
&= (\delta_{lj}\delta_{im} - \delta_{lm}\delta_{ij})x_m p_l - (\delta_{in}\delta_{kj} - \delta_{ij}\delta_{kn})x_k p_n \\
&= \delta_{lj}\delta_{im}x_m p_l - \delta_{ij}\vec{x} \cdot \vec{p} - \delta_{in}\delta_{kj}x_k p_n + \delta_{ij}\vec{x} \cdot \vec{p} = x_i p_j - x_j p_i
\end{aligned}$$

$$\epsilon_{ijk}L_k = \epsilon_{ijk}\epsilon_{klm}x_l p_m = \epsilon_{kij}\epsilon_{klm}x_l p_m = (\delta_{il}\delta_{jm} - \delta_{im}\delta_{jl})x_l p_m = x_i p_j - x_j p_i$$

Solution 25.5 set 6.

Using μ we can write

$$\vec{r}_1 = \frac{\mu}{m_1} \vec{r}, \quad \vec{r}_2 = -\frac{\mu}{m_2} \vec{r}.$$

Therefore,

$$m_1 \vec{r}_1 \times \vec{v}_1 + m_2 \vec{r}_2 \times \vec{v}_2 = m_1 \frac{\mu}{m_1} \vec{r} \times \vec{v}_1 - m_2 \frac{\mu}{m_2} \vec{r} \times \vec{v}_2 = \mu \vec{r} \times \vec{v}.$$