ADVANCED DYNAMICS — PHY 4241/5227

HOME AND CLASS WORK – SET 2

(January 12, 2010)

(9) Read Chapter 6.1 to 6.3 of M&T. Due January 20 before class.

(10) Consider a particle of mass $m = 1$, moving from $x_1 = 0$ at time $t_1 = 0$ to $x_2 = 1$ at time $t_2 = \pi/2$, under the influence of a one-dimensional harmonic potential of the form $V(x) = x^2/2$.

1. Using Newton's equations of motion, obtain the time-dependent motion of the system; *i.e.,* solve for $x(t)$. Compute the action for this exact path.
2. Using an approximate linear path of the form $x(t) = a + bt$, compute the action for this path and compare it with the value obtained before. Hint: Make sure that the path is consistent with the boundary conditions.
3. Assume that the action result of (2.) is in units $J \cdot s$ and express it in multiples of $\hbar = 1.05 \times 10^{-34}\, J \cdot s$.

Due January 15 before class (10 points).

(11) Write down and solve the Euler-Lagrange equations for

1. The one-dimensional harmonic oscillator $T = m\dot{x}^2/2$, $U = k\,x^2/2$.
2. An unphysical system with $T = 0$, $U = x^2/2$. Is the action a minimum?
3. The three-dimensional harmonic oscillator $T = m\vec{v}^{\,2}/2$, $U = k\,\vec{r}^{\,2}/2$.
4. Circular motion without friction $T = m\,l^2\,\dot{\theta}^{\,2}/2$, $U =$ const.

Due January 13 and 15 in class (8 points).

(12) Consider the spherical pendulum (a point mass m on the surface of a sphere of radius R under the influence of gravity $-g\hat{z}$).

1. Write down the Lagrange function using spherical coordinates.
2. Find the Euler-Lagrange equations.
3. Calculate the special solutions for $\theta =$ constant. Describe this motion.

Due January 22 before class (6 points).