ADVANCED DYNAMICS — PHY 4241/5227

HOME AND CLASS WORK – SET 4

(January 27, 2010)

(16) Calculate explicitly

$\delta_x L$ for the Lagrangian of two harmonically bound particles in 1D

$$L = \frac{m_1}{2}(\dot{x}_1)^2 + \frac{m_2}{2}(\dot{x}_2)^2 - k\,(x_1 - x_2)^2 \ .$$

Is there a conservation law? Due February 3 in class (2 points).

(17) Continue with the double pendulum from assignment 13.

1. Use eigenvalues $\omega_\pm$ as given in the posted solution and calculate corresponding eigenvectors in the form
$$\begin{pmatrix} 1 \\ \psi_{0\pm} \end{pmatrix} \ .$$
2. Are these eigenvectors orthogonal?
3. Write down the general solution.
4. Express the integration constants of your solution through the angular positions and velocities at time $t = 0$, denoted by ϕ_0, $\dot{\phi}_0$, ψ_0, $\dot{\psi}_0$.
5. Use $\sqrt{l/g}$ as time unit and plot the solutions $\phi(t)$ and $\psi(t)$ up to $t = 50\sqrt{l/g}$ for initial conditions $\phi_0 = 0,\ \dot{\phi}_0 = 1,\ \psi_0 = 0,\ \dot{\psi}_0 = -1$.

Due February 1 before class (10 points).

(18) Consider the spherical pendulum of mass m again.

(a) Use the Legendre transformation

$$H(\theta, \phi, p_\theta, p_\phi) = \dot{\theta} p_\theta + \dot{\phi} p_\phi - L(\theta, \phi, \dot{\theta}, \dot{\phi}) \ ,$$

to construct the Hamiltonian of the system and show that it is identical to the energy $E = T + V$.

(b) Write down Hamilton's equations of motions for the system and identify a conserved quantity.

Due February 3 before class (6 points).

(19) Calculate explicitly

$\delta_{x^i} L$ $(i = 1, \ldots, 3)$ for the Lagrangian of the 3D Kepler problem

$$L = \frac{m_1}{2}(\dot{\vec{x}}_1)^2 + \frac{m_2}{2}(\dot{\vec{x}}_2)^2 - \frac{g\, m_1 m_2}{|\vec{x}_1 - \vec{x}_2|}$$

Are there associated conservation laws? Due February 3 in class (4 points).