ADVANCED DYNAMICS — PHY 4241/5227

HOME AND CLASS WORK – SET 6

(February 8, 2010)

(22a) Consider the Lagrangian

$$L \;=\; \frac{1}{2}\, m\, \dot{x}^2 - \frac{1}{2}\, k\, x^2 \ .$$

1. Name the system which this Lagrangian describes.
2. Is the energy conserved? Y/N.
3. Is the (linear) momentum conserved? Y/N.

Due February 8 in class (3 extra points).

(22b) Let the interaction of two point particles be described by a potential which depends only on their distance:

$$\mathcal{L} = \frac{m_1}{2}\vec{v}_1^{\,2} + \frac{m_2}{2}\vec{v}_2^{\,2} - U(r)\,, \quad r = |\vec{r}|\,,\ \vec{r} = \vec{r}_1 - \vec{r}_2\ .$$

1. Is the energy of this system conserved (with reason) (2 points)?
2. Define the center of mass vector $\vec{R} = \vec{R}(t)$ by $M\dot{\vec{R}} = m_1\dot{\vec{r}}_1 + m_2\dot{\vec{r}}_2$, where $M = m_1 + m_2$ is the total mass, and find the general solution for $\vec{R}(t)$ (2 points).
3. Use the special solution $\vec{R} = (m_1\vec{r}_1 + m_2\vec{r}_2)/M$, to express $\vec{r}_1$ and $\vec{r}_2$ through $\vec{R}$ and $\vec{r}$ (3 points).

   The center of mass (cm) systems is defined by $\vec{R}(t) = 0$. Show the following equalities in the cm system:

4. $T_{\rm cm} = m_1\,\vec{v}_1^{\,2}/2 + m_2\,\vec{v}_2^{\,2}/2 = \mu\,\vec{v}^{\,2}/2$ with $\vec{v} = \dot{\vec{r}}$ and μ the reduced mass. Express μ through m_1, m_2 and M (3 points).
5. $\vec{L} = \vec{r}_1 \times \vec{p}_1 + \vec{r}_2 \times \vec{p}_2 = \mu\,\vec{r} \times \vec{v}$ (2 points).

Due February 10 in class.

Read M&T chapter 8.1 to 8.7. Due February 15.